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EDDY DIFFUSIVITY AND THE FOUR-THIRDS LAW IN NEAR-SHORE (COASTAL WATERS)

James E. Foxworthy

University of Southern California
Los Angeles, California

1968

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EDDY DIFFUSIVITY AND THE FOUR-THIRDS LAW IN NEAR-SHORE (COASTAL WATERS)

JAMES E. FOXWORTHY

REPORT 68-1

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EDDY DIFFUSIVITY AND THE FOUR-THIRDS LAW IN NEAR-SHORE COASTAL WATERS

Details of illustrations in
this document may be better
studied on microfiche

James E. Foxworthy

Report 68-1

Abstract:

During the period from 1963 through 1968 a series of instantaneous and continuous release dye experiments were conducted in the near-shore coastal waters off southern California. The results of these experiments have been interpreted in terms of two- and three-dimensional apparent coefficients of eddy diffusivity. During the course of this investigation, little evidence was found to substantiate the applicability of the so-called four-thirds law relating the instantaneous apparent diffusivity coefficients and the approximate eddy scale. A comparison of the mean diffusivity coefficients for these experiments showed that the scale-dependency was variable being related to the time of observation, rates of diffusion, and definition of eddy scale. In addition, the mean vertical diffusivity coefficients were found to be related to the mean wind speed and water column stability. The mean horizontal coefficients were found to be independent of mean wind speed, current speed, and stability. These findings cast doubt on the direct applicability of the four-thirds law in the design of submarine waste outfalls or in predicting the behavior of other low density surface fields.

Acknowledgements

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The work could not have been completed without the full cooperation of representatives of many state and local agencies. A complete listing of all those involved is not possible at this time, however, the writer would like to acknowledge the invaluable assistance of the following: Mrrs. R. Dennis Hogle and A.E. Clancy (City of Santa Barbara); Hugh Clark, Jr. and Stanley Pateman (City of Oxnard); David Sullivan (City of Ventura); Paul Brown, Ted Dunn and Robert Galloway (Orange County Sanitation Districts); T.J. Gannon (California State Department of Public Health); and Ken Jones (California State Water Quality Control Board).

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Introduction

Recently there have been an increasing number of contributions to scientific and technological literature regarding various theoretical concepts of turbulent eddy diffusion in the ocean and other large bodies of water. However, to date there has been a paucity of experimental data collected under field conditions from which to corroborate the various theoretical predictions. In particular, there is little information available on multi-dimensional diffusivity coefficients and the significance of vertical diffusion in the overall dispersion process. Horizontal diffusion has been studied extensively; however, there appears to be some ambiguity, especially in the technological literature, regarding the manner in which the functional relationship between observed coefficients of horizontal diffusivity and associated eddy scale have been established and interpreted.

With the advent of seaworthy continuous flow fluorometers and relatively conservative fluorescent tracer dyes, it is now possible to examine the mechanics of turbulent diffusion in the ocean in much greater detail than was possible only a few years ago. This report presents the results of a series of experiments designed to investigate multi-dimensional turbulent eddy diffusion of a fluorescent dye tracer released from instantaneous and continuous point- and large-scale sources. The majority of these experiments were conducted within surface waste fields resulting from the discharge of domestic and industrial wastes through submarine outfalls located off the coast of southern California. Surface fields formed by the discharge of relatively low salinity waste waters are usually highly stratified compared to the surrounding ocean water. By conducting

experiments inside and outside such surface fields, it was possible to assess the effects of density stratification as well as other environmental phenomena on observed rates of horizontal and vertical diffusion.

Geographic Areas

The bulk of the dye patch and continuous point source plume experiments were conducted within surface waste fields generated by the discharge of waste water from the Orange County, California treatment plant. The waste effluent from this plant receives primary treatment in the form of screening and sedimentation before discharge through a 2000m long submarine outfall which terminates in approximately 15 meters of water.

Additional outfall sites were selected for full-scale experiments. These outfalls are operated by the cities of Ventura, Oxnard and Santa Barbara, California. In each case partially treated waste water is discharged in the ocean through a submarine outfall terminating in relatively shallow water (10 to 15m depth) at a distances of from 975 to 2070m offshore (See Appendix-Table 4). The surface fields generated by these relatively small waste flows were considerably smaller in size than that observed at Orange County.

Several oceanic areas were selected as background sites for control experiments in which little if any waste water was present. These areas were located within a 25 metric mile radius of the Orange County site.

Experimental Procedures

The following is a brief summary of the experimental methods and equipment used in this investigation. A detailed description was presented by the Allan Hancock Foundation (1965).

Instantaneous release - dye patch studies.

Rhodamine-B dye solution was discharged instantaneously from a small launch at a point near the origin of a surface waste field. The dye concentration distribution in the vertical plane was first obtained by locating a small launch in the zone of most intense dye concentration and pumping water samples from known depths through a continuous flow fluorometer. With the depth of maximum dye concentration established (usually at or near the surface), the concentration distributions along the horizontal axes of the patch were determined by criss crossing the patch while towing a small sampler set at the previously determined depth of maximum dye concentration. This sampling procedure was repeated at regular intervals over a period of several hours. Using these techniques it was possible to obtain reasonably accurate records of the three-dimensional profiles of dye concentration as related to average diffusion time.

Data necessary for the determination of the vertical density structure were obtained by use of a Frautchi water sampler and a thermistor probe. Current velocities were obtained by radar tracking of the center of the dye patches and drogues released at the head of the waste plume. In later experiments a current meter with direction sensor was also employed. Wind data was obtained from continuous recording anemometers located at nearby shore stations and on board the main research vessel, VELERO IV.

Continuous release - dye plume experiments.

In the point-release experiments, a small skiff containing a dye solution tank and pump was anchored near the head-end of the waste plume. Dye solution was released at a constant rate through a small plastic tube located near the water surface. Shortly after the start of the test, a series of surface floats were anchored at known locations along the resulting dye plume. All lateral and vertical sampling of the plume was conducted at these locations. The sampling procedure followed was to first determine the lateral dye concentration distribution by criss crossing the plume several times in the vicinity of a particular float, then move into the plume and sample vertically at the same location. The procedure was repeated at various points from the head to the tail of the plume. The approximate distribution of maximum dye concentration along the longitudinal axis of the plume was obtained by towing the sampler up and down the dye plume at a known launch speed. The time necessary to make one complete sampling run of the entire plume averaged about one hour. This procedure was repeated several times during each experiment.

The continuous release experiments were later extended to include studies of dispersion from large-scale sources. In these experiments dye was injected directly into the waste effluent before discharge to the ocean. The entire waste plume was then sampled by the methods described above for the point-release studies. A more detailed account of these studies was given by Foxworthy and Kneeling (1968).

Diffusivity Coefficients

The following is a brief review of the methods commonly em-

ployed to estimate diffusivity coefficients and related scale effects as they apply to the dispersion of tracer dyes in the marine environment. Since one of the principal aims of this paper is to discuss why certain empirical methods previously employed to describe the diffusivity-scale relationship may have been misleading, the various theoretical concepts of turbulent diffusion in ocean and lake environments will be referred to sparingly. The recent comprehensive reviews by Okubo (1962 a, b), Bowden (1965), and Csanady (1963), cover the subject in greater detail.

When dealing with problems involving the relative diffusion of dye tracers, e.g., diffusion about the center of mass of a dye patch, it is customary to define an "apparent" or "effective" instantaneous coefficient of diffusivity by the equation

$$K = \frac{1}{2} \frac{d}{dt} \sigma^2 \qquad (1)$$

where σ^2 is the mean variance or second movement of the tracer concentration distribution in a given coordinate direction as a function of dispersion time t. The term "apparent" or "effective" is used to indicate that the net dispersion of material is the result of the interaction of molecular and turbulent eddy motion, current shear, density effects etc.

Equation (1) has also been used to estimate relevant diffusivity coefficients for the case of dispersion from a continuous fixed source in a steady current. Thus, substituting $x/\bar{U}$ for t in (1) gives

$$K = \frac{\bar{U}}{2} \frac{d}{dx} \sigma^2 \qquad (2)$$

where $\bar{U}$ is the mean current speed and σ^2 is expressed, in this case, as a function of distance.

In order to compare the data obtained in dye tracer studies with current theories of turbulent eddy diffusion it is usually assumed that the variance σ^2 is proportional to the so-called mean square separation ξ^2 of two particles whose relative motion is brought about by turbulence. In the case of the relative diffusion of dye patches, the dimensional predictions proposed by Batchelor (1950) can be used to describe the temporal distribution of σ^2. According to Batchelor the relative diffusion of a tracer substance proceeds in three distinct phases depending on diffusion time. Thus, for small times and when the size of the patch is smaller than the so-called inertial sub-range of eddy sizes (Okubo (1962 a)) we have

$$\sigma^2 - \sigma_0^2 = K_1 (\epsilon \sigma_0)^{2/3} t^2 \qquad (3)$$

where σ_0^2 is the variance in a given coordinate direction at t = o, ϵ is the rate of energy dissipation per unit mass, and k is a constant. For large times such that the influence of σ_0^2 on the diffusion process is small, yet the scale of the process is still within the inertial sub-range, the variance is given by

$$\sigma^2 = k_2 \epsilon t^3 \qquad (4)$$

Eventually at relatively large values of time such that diffusion is controlled by eddies on a scale larger than the inertial sub-range, the asymptotic phase is reached and

$$\sigma^2 = k_3 t \tag{5}$$

The time t_1 separating the first and second diffusion phases is given by Batchelor (1950) as

$$t_1 \doteq \frac{\sigma_o^{2/3}}{\epsilon^{1/3}} \tag{6}$$

Gifford (1959) proposed that Batchelor's predictions for relative diffusion also apply to the diffusion of a continuous plume of material in a steady current. Gifford assumed a continuous plume to be constructed of an infinite array of overlapping "puffs" or "disk elements" each with a Gaussian material concentration distribution as released from a fixed source. Within the limits of these assumptions, equation (3) through (6) would be expected to describe the mean dispersion of tracer dye along the lateral (cross-plume) and vertical axes of a steady continuous plume.

Recently, the effect of shear due to non-homogeneous currents on the rate of growth of both dye patches and continuous dye plumes has been investigated by Bowden (1965), Okubo (1966), and Csanady (1966). These authors show that a variety of growth rates are possible depending on existing current and boundary conditions. In certain cases the predicted σ^2 - t relationships are similar to those presented by Batchelor.

It is easy to show from the above theoretical predictions that instantaneous values of K computed from equation (1) or (2), will vary with time (or distance) unless $\sigma^2 \propto t$. This limiting condition holds for both diffusion of a tracer dye in a homogeneous or

non-homogeneous (shearing) current field.

In many past investigations of oceanic diffusion it was possible to obtain a few or even single estimates of σ^2 and t from which to estimate pertinent diffusivity coefficients. In many of these studies the coefficients were assumed to be constant which implied that the asymptotic diffusion phase had been reached, or in other words the diffusion process followed Fick's Law (Fick (1855)). For these conditions integration of equation (1) yields

$$K = \frac{\sigma_2^2 - \sigma_1^2}{2\,(t_2 - t_1)} \tag{7}$$

In the event the diffusion process is not within the asymptotic phase, equation (7) actually yields an approximate mean coefficient for the time interval involved. This can be seen by the definition of the mean value given by

$$K = \frac{1}{t}\int K dt = \frac{1}{t}\int d\,\sigma^2 \tag{8}$$

Substituting equation (1) for K into equation (8) and integrating between the same limits as above yields equation (7). Thus, it is apparent that whenever equation (7) is used to estimate K, the result will be a true mean value unless the asymptotic diffusion phase has been reached; in this case the instantaneous and mean values of K are equal.

Equations similar to (7) have long been used to estimate horizontal, and in some cases, vertical diffusivity coefficients. However, the wide range in reported values led earlier investigators to conclude that the coefficients were not constant but related to

the scale of the diffusion phenomena. Thus, following the earlier work of Richardson (1926) in atmospheric diffusion, it was proposed that

$$K \propto L^m \tag{9}$$

where L is a measure of the mean eddy size promoting diffusion and m is a somewhat arbitrary exponent. When m = 4/3, equation (9) reduces to Richardson's well known 4/3-law.

As discussed by Orlob (1959), the most difficult task faced in attempting to establish an empirical relationship such as (9), has been the determination of a definitive measure of the scale of diffusion. For reasons based more on convenience than theory the mean standard deviation of the diffusing material concentration distribution for a specified time interval, that is,

$$\overline{\sigma} = \frac{\sigma_2 + \sigma_1}{2} \tag{10}$$

has become one of the most widely used estimates of mean eddy scale. However, according to the statistical theories of turbulence, such an approximation is only justified when the magnitude of $\overline{\sigma}$ is equal to or smaller than the characteristic length scale of turbulence for the particular oceanic area under investigation. Once the asymptotic diffusion phase is reached the coefficient of diffusivity becomes constant and is proportional to this characteristic length scale; however, $\overline{\sigma}$ continues to increase with time and hence can no longer be used as an estimate of scale. Obviously knowledge of the magnitude of the length scale of turbulence in

various oceanic areas would avoid this difficulty. Unfortunately, to date there appears to be no practical method available for the accurate determination of this parameter.

Assuming $\overline{\sigma}$ to be a rough estimate of scale, at least in cases where K is not constant, and substituting into (9) yields the empirical relationship

$$K \propto (\overline{\sigma})^{m} \tag{11}$$

or when m = 4/3

$$K \propto (\overline{\sigma})^{4/3} \tag{12}$$

A theoretical relationship similar to (12) can be derived for relative diffusion of dye patches based on the similarity theory of homogeneous turbulence (Okubo, (1962 a)). Thus, for diffusion within the inertial sub-range of eddy sizes

$$\sigma^2 \propto \epsilon t^3 \tag{4}$$

Substituting this relationship for σ^2 into equation (1), differentiating and eliminating t yields

$$K \propto \epsilon^{1/3} \sigma^{4/3} \tag{13}$$

and with a constant rate of energy dissipation

$$K \propto (\sigma)^{4/3} \tag{12}$$

Consequently, the 4/3-law relating the instantaneous diffusivity coefficients and approximate eddy scale would be expected to apply only within the inertial sub-range of eddy sizes where $\sigma^2 \propto t^3$.

Until recently there have been few experiments conducted over a sufficiently long time period to completely define the variance vs. time relationship for oceanic turbulence. Therefore, it has not been possible to verify directly the theoretical predictions regarding mean square diffusion or the scale-dependency of diffusivity coefficients. Consequently, many past investigators have attempted to establish the scale-diffusivity relationship by synthesizing data from a variety of individual experiments. Pearson (1956), Okubo (1962 a), and Weigel (1964) presented graphical summeries of all previously reported values of diffusivity coefficients as a function of eddy scale. Although the combined data represented many definitions of the variables, these summaries appeared to substantiate a 4/3 relationship for an arbitrary scale range of 10^3 to 10^9 cm. Orlob (1959) and Okubo (1962 a) pointed out that these results may have been somewhat fortuitous. To these authors it was difficult to conceive of an inertial sub-range extending from 10^3 to 10^9 cm in length, even in the ocean.

In most of the experiments described in this paper, it was possible to obtain sufficient data to establish empirical relationships between σ^2 and t in both horizontal and vertical coordinate directions for observation times ranging from about two to seven hours. From these data it was possible, using statistical inference techniques, to determine if the 4/3-law described the relationship between instantaneous values of K and σ for individual dye patches or plumes, or as in past investigations, between the corresponding mean values of these variables for all experiments.

Experimental Results

Instantaneous Coefficients. In many of these experiments the instantaneous concentration profiles for dye patches and mean profiles for dye plumes were found to follow a Gaussian law. Values of the variance, σ^2, for these profiles were calculated by curve fitting techniques described by the Allan Hancock Foundation (1965), and Diaschisin (1963). In most experiments there appeared to be a strongly correlated linear relationship between the computed values of σ^2 and diffusion time (or distance) when these variables were plotted on log-log paper. An example of the results obtained from a dye patch study conducted within a surface waste field over the Orange County outfall is shown in Figure 1. Table 1 (appendix) is a summary of the correlation coefficients and corresponding levels of significance for the least squares regression lines shown in Figure 1. In all cases there was a highly significant correlation between the variables. Since the number of data points was small, Students' t-test was used to ascertain if the calculated slope of the regression lines were significantly different from the values predicted by the various theories of diffusion discussed previously. The results of this analysis are summarized in the last two columns of Table 1. These data indicate that the computed value of the exponent on t, for the longitudinal growth of the patch was not significantly different from the theoretical value of 2.0 at the arbitrarily selected 1% level of significance. Similarly, the exponent for the lateral and vertical rates of growth were not significantly different from the theoretical values of 2.0 or 1.0. The significant fact brought out in this analysis was that none of the coordinate variances could be shown statistically to increase with time

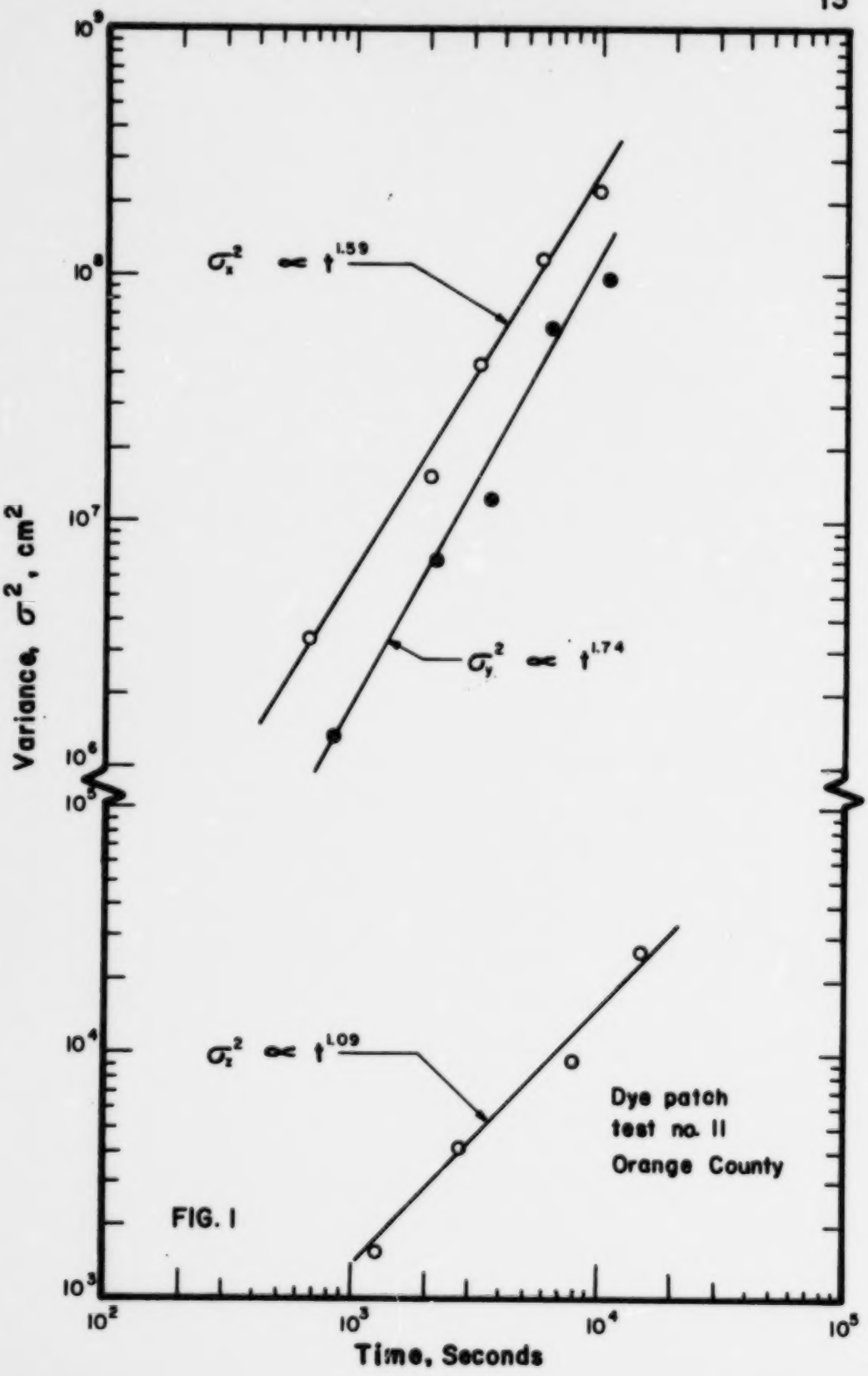

$\sigma_x^2 \propto t^{1.59}$
$\sigma_y^2 \propto t^{1.74}$
$\sigma_z^2 \propto t^{1.09}$
Dye patch
test no. II
Orange County
Variance, σ^2, cm^2
Time, Seconds
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9

FIG. 1

cubed for the period of observation indicated. Similar results were observed in most dye patch and continuous dye plume experiments conducted within the various surface waste fields and background areas as further illustrated by the data shown in Figures 2 and 3.

Theoretically, the predictions regarding the rate of growth of a dye patch are in terms of the mean values of σ^2 determined over a large number of identical experiments. The variances shown in Figures 1 and 2 are essentially instantaneous values for two individual experiments. Estimates of the true mean variance are difficult to obtain by the sampling procedures employed in the dye patch experiments. It was possible, however, to compare instantaneous values of the lateral variances, for example, from several experiments conducted under similar oceanographic conditions and assume these data were proportional to the true mean values. Such a comparison is shown in Figure 4 for a series of six independent dye patch experiments. The t-test applied to these data showed that the indicated slope of the least squares trend line is different from 3.0 at the 1% level of significance. Hence the analysis for the pooled data yielded essentially the same results as for individual dye patch and plume experiments, that is, no statistical evidence of a $\sigma^2 \propto t^3$ relationship could be found.

It should be noted that no initial values of the variance, σ_o^2, were obtained for the data shown in Figures 1 through 4 because of difficulties involved in sampling during the first few minutes after dye release. Reliable estimates of the variance of the horizontal dye distribution were made from aerial photographs and visual field estimates and ranged from 10^3 to 10^4 cm^2. In most

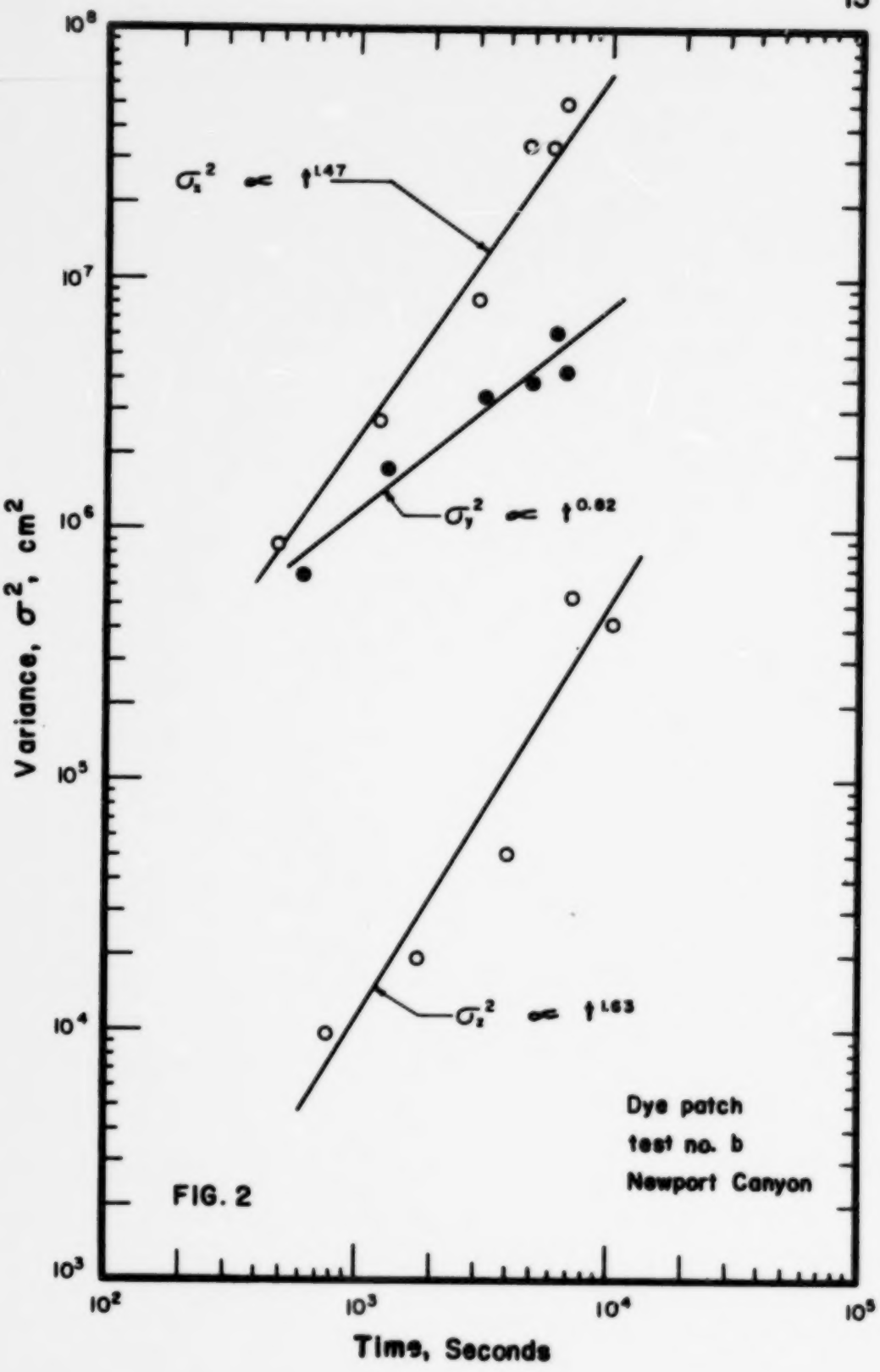

10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^3
10^4
10^5
Variance, σ^2, cm^2
Time, Seconds
$\sigma_x^2 \propto t^{1.47}$
$\sigma_y^2 \propto t^{0.82}$
$\sigma_z^2 \propto t^{1.63}$
Dye patch
test no. b
Newport Canyon

FIG. 2

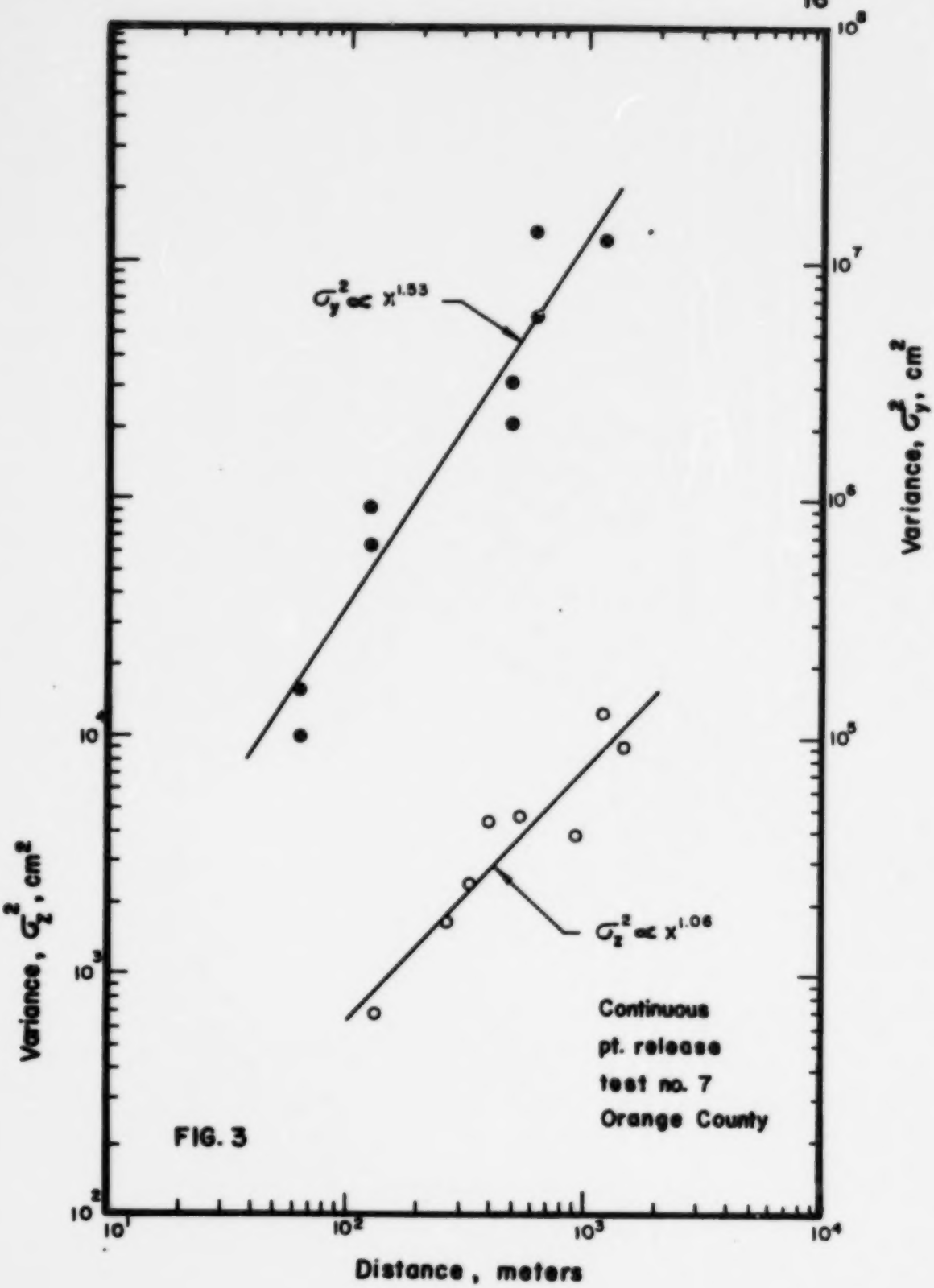
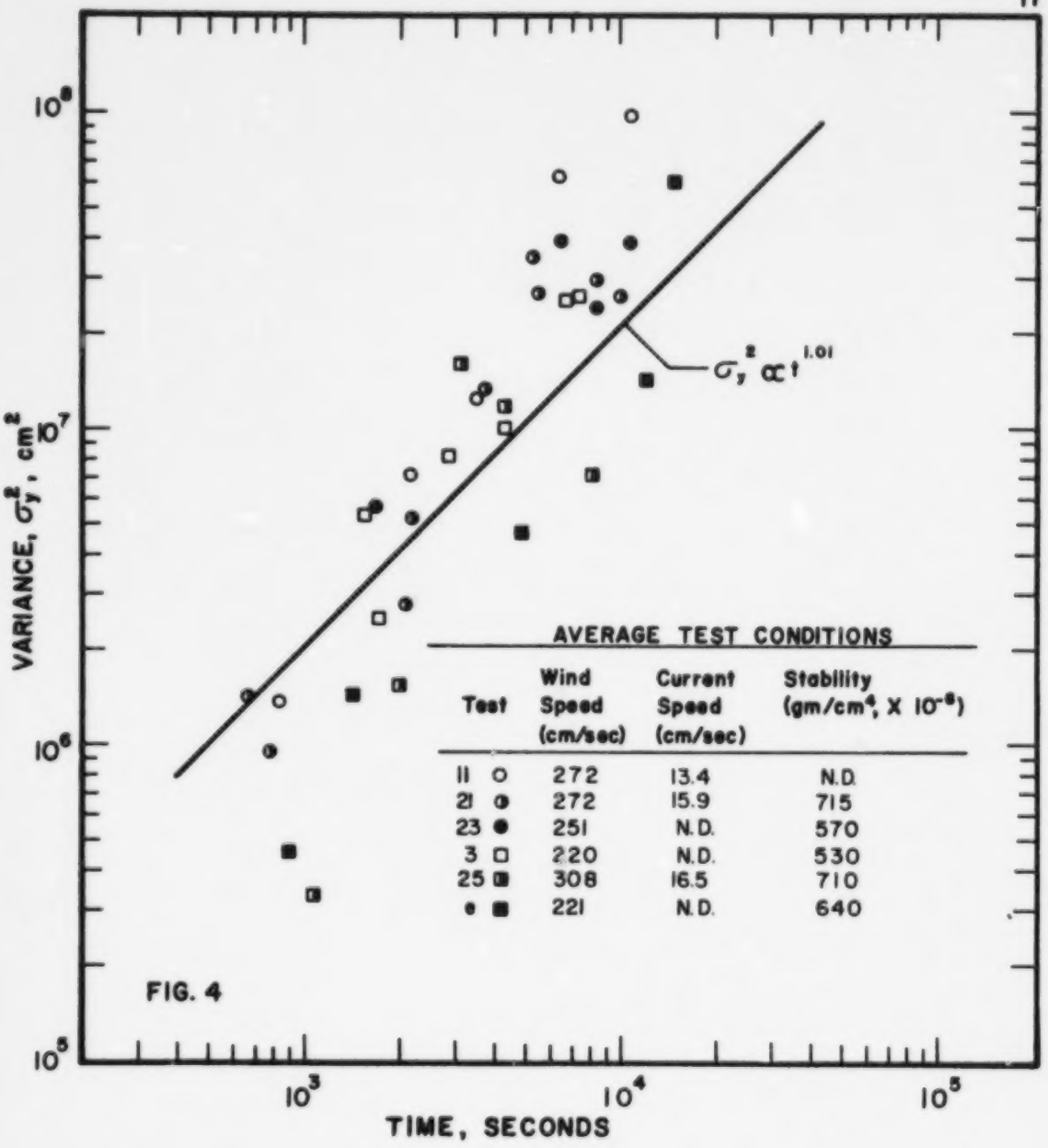

$\sigma_y^2 \propto x^{1.53}$
$\sigma_z^2 \propto x^{1.06}$
Continuous
pt. release
test no. 7
Orange County
Variance, σ_z^2, cm^2
Variance, σ_y^2, cm^2
Distance, meters
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8

FIG. 3



AVERAGE TEST CONDITIONS

Test	Wind Speed (cm/sec)	Current Speed (cm/sec)	Stability (gm/cm^4, X 10^{-6})
11 ○	272	13.4	N.D
21 ◑	272	15.9	715
23 ●	251	N.D.	570
3 □	220	N.D.	530
25 ◨	308	16.5	710
• ■	221	N.D.	640

FIG. 4

dye patch and point release plume experiments these values of σ_o^2 were used to make a crude estimate of the value of t_1 (eq. (6)) seperating the first and second diffusion phases proposed by Batchelor. These estimates of t ranged from 60 to 300 sec (cf Allan Hancock Foundation (1965)). Therefore, according to equations (4) and (5), the observed values of σ^2 in these experiments would be expected to be proportional to t or x cubed near the source with the exponent decreasing to 1.0 at large values of t or x. The fact that no $\sigma_y^2 \propto t^3$ regime was observed indicates that the maximum eddy size defining the so-called inertial subrange was much smaller than originally expected for the oceanic areas studied.

Additional evidence to substantiate the above findings is given by the composite vertical photograph shown in Figure 5. In this experiment a continuously tagged waste plume is shown approximately two hours after the dye appeared at the water surface. Visual inspection of the tagged waste field shows that the lateral boundaries of the steady segment of the plume were approximately parabolic, thus indicating a constant rate of diffusion. Repeated field measurements of the lateral and vertical dye concentration profiles were made at several cross sections within the same segment of the plume. Values of $\sigma^2 - \sigma_o^2$ for the mean variances computed from these profiles are shown plotted against distance as shown in Figure 6. Statistical analysis of these date showed that neither exponent was different from a theoretical value of 1.0 at the 1% significance level. Thus, the lateral as well as vertical rates of growth within the first 700m of the waste plume were constant and did not follow that predicted by equations (3) or (4).

These results are in accord with the previous conclusion that

Ventura Outfall
November 11, 1966

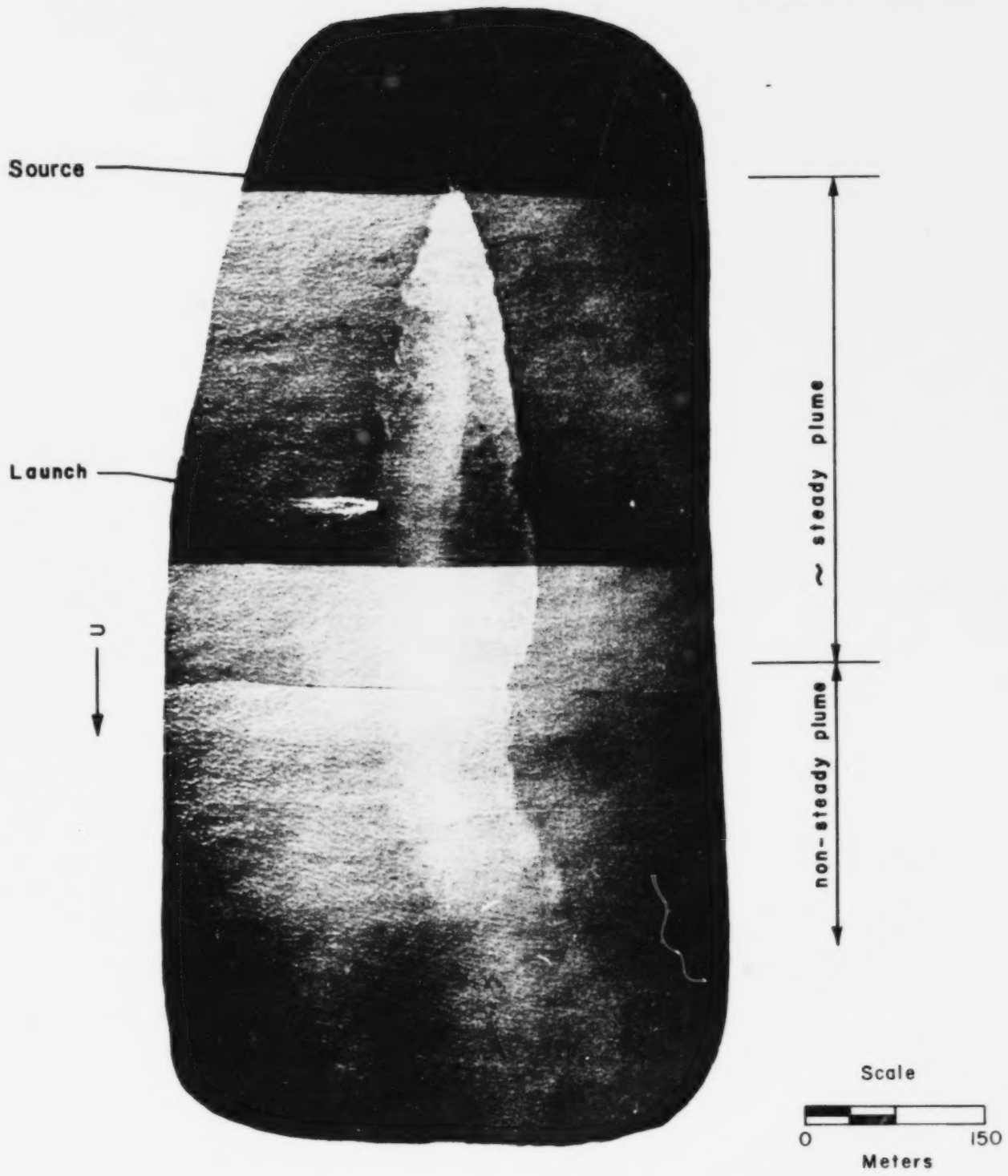


FIG. 5

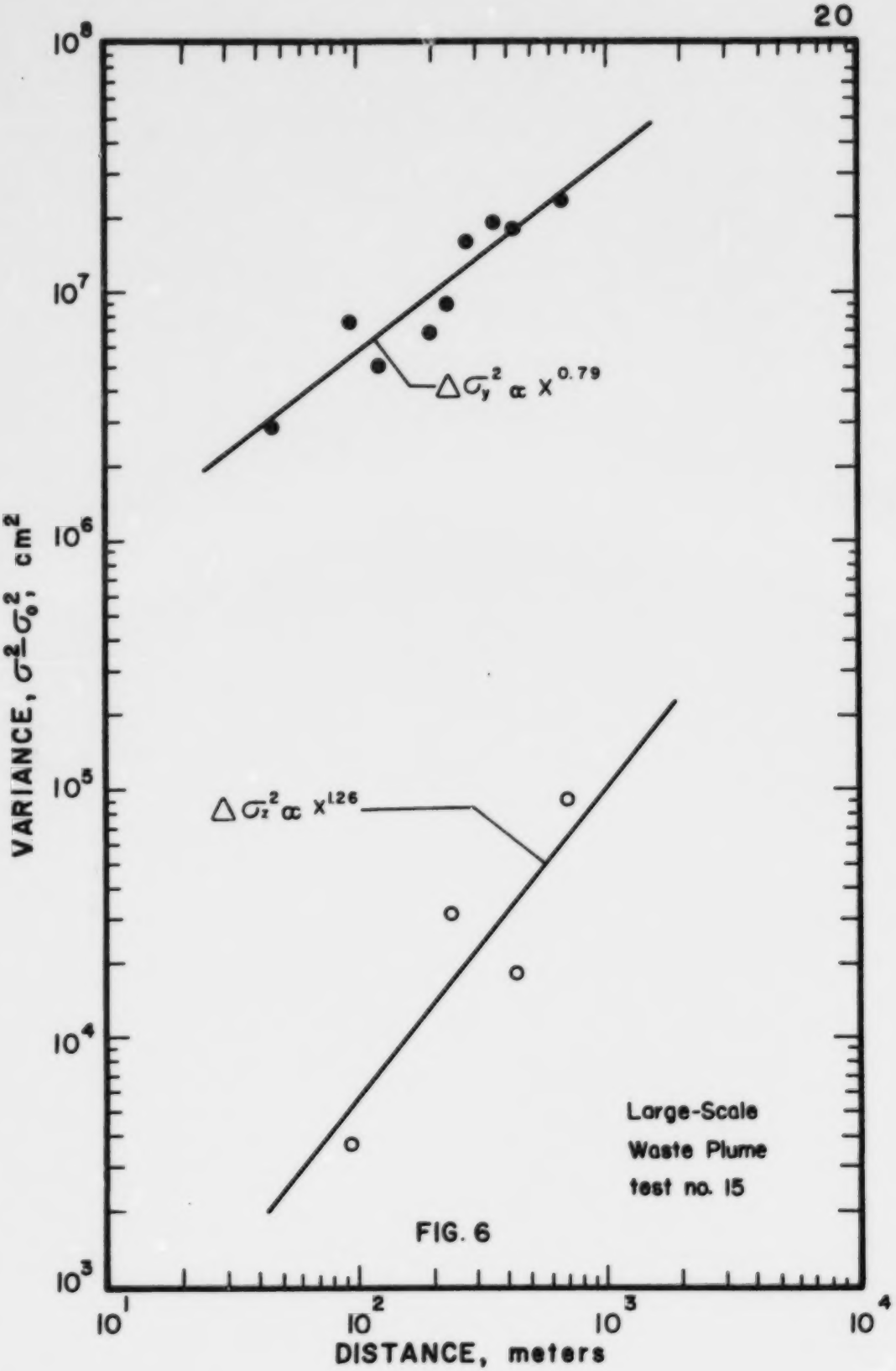

10^8
10^7
10^6
10^5
10^4
10^3
VARIANCE, $\sigma^2-\sigma_o^2$, cm^2
$\Delta\sigma_y^2 \propto x^{0.79}$
$\Delta\sigma_z^2 \propto x^{1.26}$
Large-Scale
Waste Plume
test no. 15
10^1
10^2
10^3
10^4
DISTANCE, meters

FIG. 6

the inertial subrange of eddy sizes to which the 4/3-law directly applies was probably limited to a scale smaller than that observed in these experiments.

It should be pointed out that the results and conclusions discussed above do not agree with those reported by Ichiye (1964) for diffusion of dye patches off the coast of New Jersey and over the Bahama Banks. Ichiye presented data similar to that shown in Figures 1 and 2 which appear to substantiate a t^2 and t^3-law for small and large diffusion times respectively. This apparent discrepency in experimental results could have been due to differences in oceanographic conditions characteristic of the areas studied.

Mean Coefficients. These findings led the writer to question certain previously reported verifications of the four-thirds law for oceanic diffusion. That is, if the 4/3-law relating the instantaneous values of K and σ did not hold for the individual dye patch or dye plume experiments, why should it apply (collectively) to the mean values of these variables for all experiments? In order to help answer this question the data were also analyzed by the methods employed by earlier investigators.

In most of the present dye patch experiments the dye was observed to elongate markedly in the direction of the mean current. The observed higher rates of longitudinal dispersion were assumed to be due to current shear associated with vertical velocity variations rather than true anisotropy of turbulent motion. Also, as will be discussed later, the vertical spread of dye was observed to be strongly influenced by vertical density stratification. For these reasons, the analysis was restricted initially to comparison of the mean lateral diffusivity coefficients.

Values of $\bar{K}y$ and $\overline{\sigma}_y$ for all experiments were estimated from the observed values of σ^2 near the beginning and end of each study together with equations (7) and (10). The computed values of $\bar{K}y$ were then plotted against $\overline{\sigma}_y$ as shown in Figure 7. Although there is considerable scatter of the plotted points, the correlation coefficient for these data is 0.8507 which is significant at less than the 0.1% level, indicating very high correlation. The t-test applied to the least squares relationship showed that the slope of the trend line was not different from a theoretical value of 4/3 or 2/1 at the 1% level of significance. The later finding is interesting because whenever the final values of the variance and observation time in equations (7) and (10) are much greater than the initial values we have

$$K_y \doteq \frac{\sigma_2^2}{2t} \tag{7a}$$

and

$$\overline{\sigma}_y \doteq \frac{\sigma_2}{2} \tag{10a}$$

Hence, whenever the time of observation is constant, a plot of $\bar{K}_y$ vs $\overline{\sigma}_y$ mearly expresses the dimensional relationship $\sigma^2 \propto \sigma$; therefore, an approximate 2/1 slope would be expected to result in a logarithmic plot of these variables. This fact cannot be demonstrated by the data shown in Figure 7 because of the wide range of observation times (35 to 400 minutes) encountered in these experiments. It was possible, however, to investigate the influence of observation time on the scale-diffusivity relationship by use of

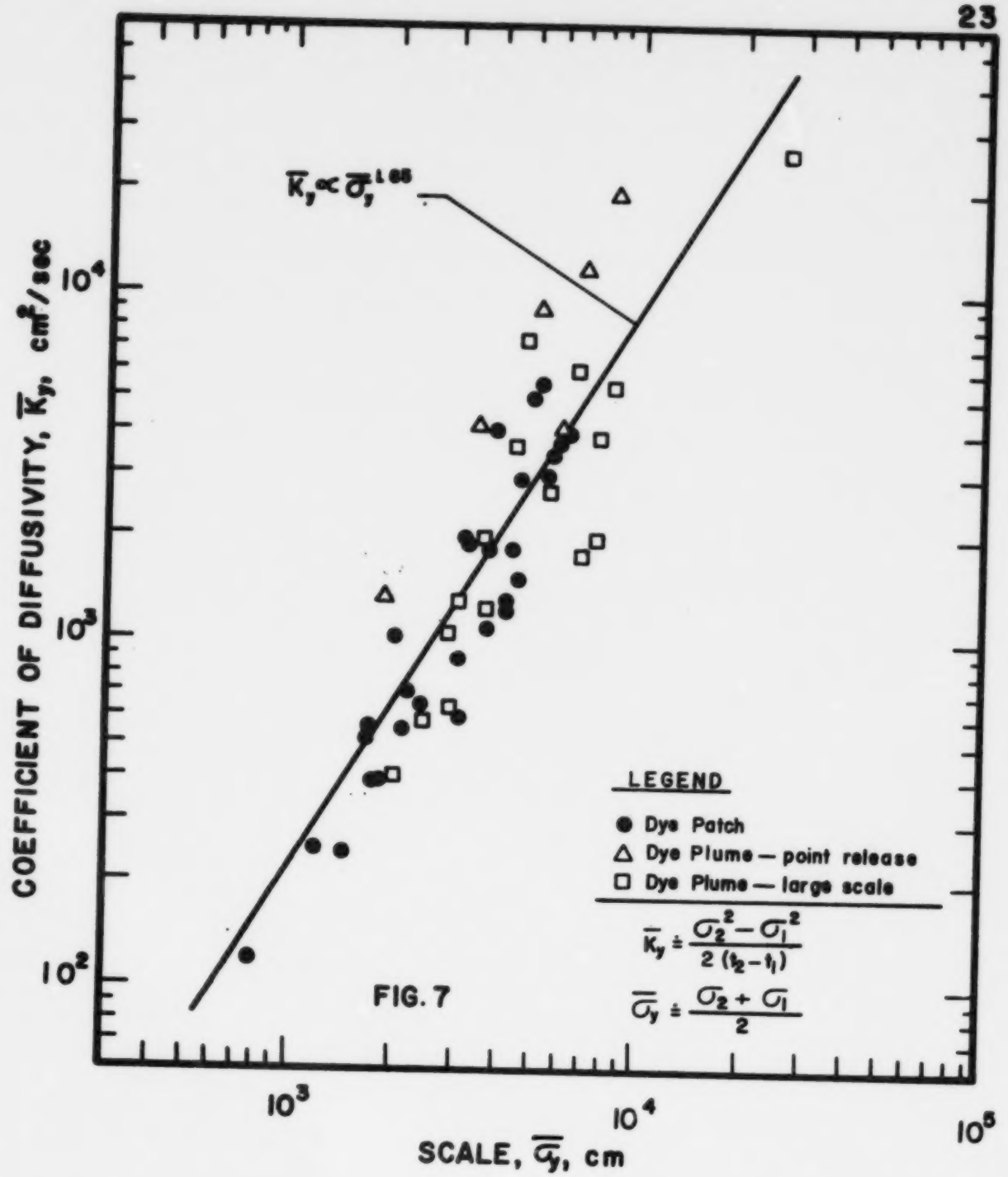

$\overline{K}_y \propto \overline{\sigma}_y^{1.66}$
COEFFICIENT OF DIFFUSIVITY, $\overline{K}_y$, cm²/sec
10^4
10^3
10^2
10^3
10^4
10^5
SCALE, $\overline{\sigma_y}$, cm
LEGEND
Dye Patch
Dye Plume — point release
Dye Plume — large scale
$\overline{K}_y \doteq \frac{\sigma_2^2 - \sigma_1^2}{2(t_2 - t_1)}$
$\overline{\sigma_y} \doteq \frac{\sigma_2 + \sigma_1}{2}$

FIG. 7

the empirically derived equations for σ_y^2 as shown for example in Figures 1 and 2. Equation (8) for $\bar{K}_y$ was solved using the empirical σ_y^2-t relationships and an arbitrarily assumed observation interval of 200 minutes. The associated mean eddy scale for this time interval was estimated from the equation for the mean value of σ_y given by

$$\bar{\sigma}_y = \frac{1}{t}\int_0^t \sigma_y \, dt \qquad (14)$$

where σ_y is the value of the standard deviation as derived from the empirical equations for σ_y^2 in terms of diffusion time. The results of this analysis are shown in Figure 8. It is evident that there is much less scatter and higher correlation (r = 0.9688) for these data than that shown in Figure 7. The t-test applied to these data indicated that the slope of the trend line is not different from 2/1 but is different from 4/3 at the 1% significance level. Thus, it appears that whenever the observation or diffusion time is constant the scale-dependency of mean diffusivity coefficient as these variables are defined above, is constant and given approximately by $\bar{K}_y \propto \left(\bar{\sigma}_y\right)^2$.

This later point is further emphasized by the data presented in Figure 9 in which the combined mean horizontal and vertical coefficients computed from equation (7) for all dye patch and plume experiments are plotted against $\bar{\sigma}$ computed from equation (10). Although these data were collected under highly variable environmental conditions, e.g., wind speed, density stratification, current shear, etc., there is a strong correlation between the variables (r = 0.9667) and $\bar{K}$ was found to be proportional to $\bar{\sigma}$ to the 1.98 power. As in the case of the data shown in Figure 7, the

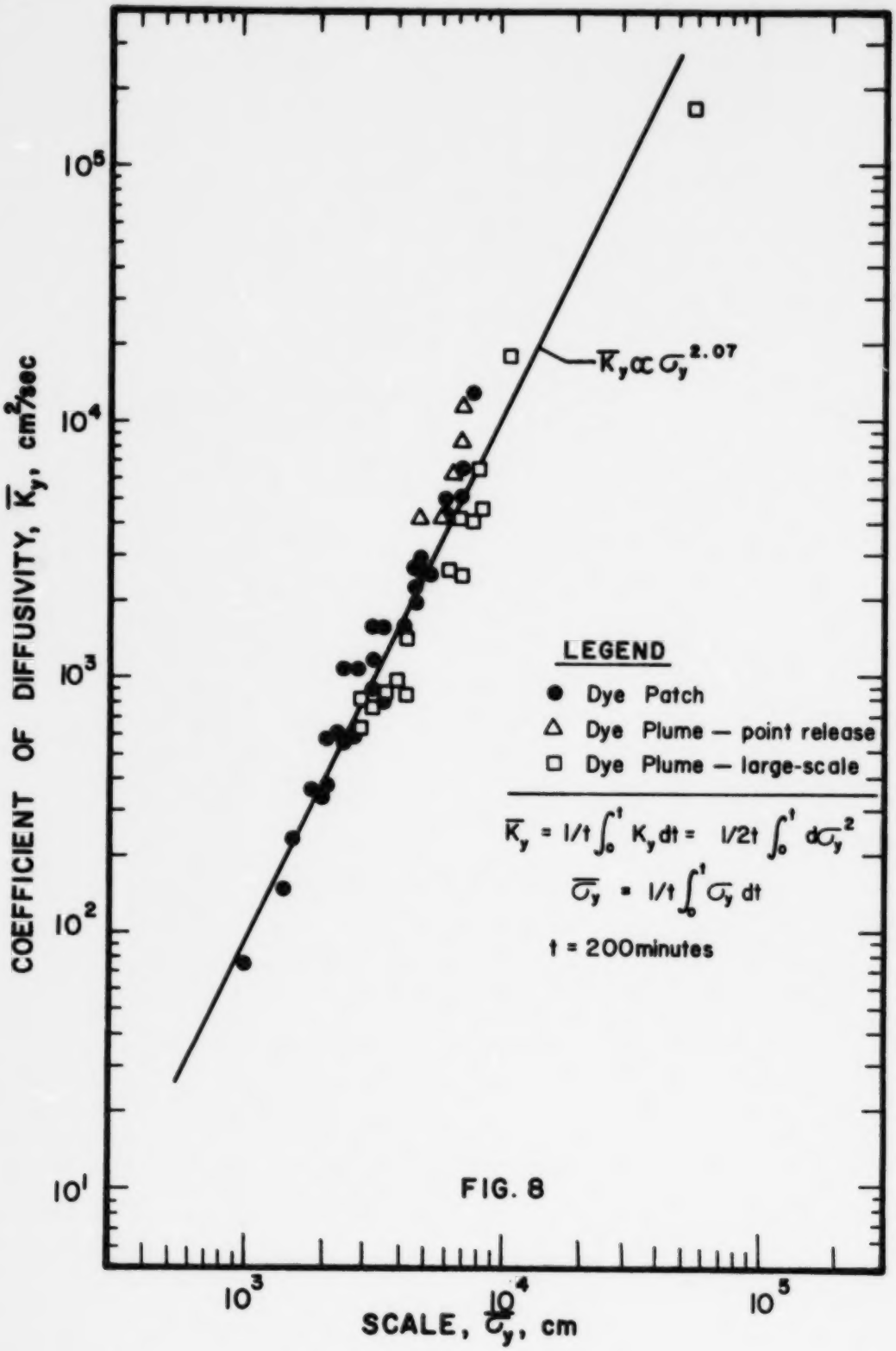

COEFFICIENT OF DIFFUSIVITY, $\bar{K}_y$, cm²/sec
SCALE, $\bar{\sigma}_y$, cm
10^1
10^2
10^3
10^4
10^5
$\bar{K}_y \propto \sigma_y^{2.07}$
LEGEND
Dye Patch
Dye Plume — point release
Dye Plume — large-scale
$\bar{K}_y = 1/t \int_0^t K_y dt = 1/2t \int_0^t d\sigma_y^2$
$\bar{\sigma}_y = 1/t \int_0^t \sigma_y dt$
t = 200minutes

FIG. 8

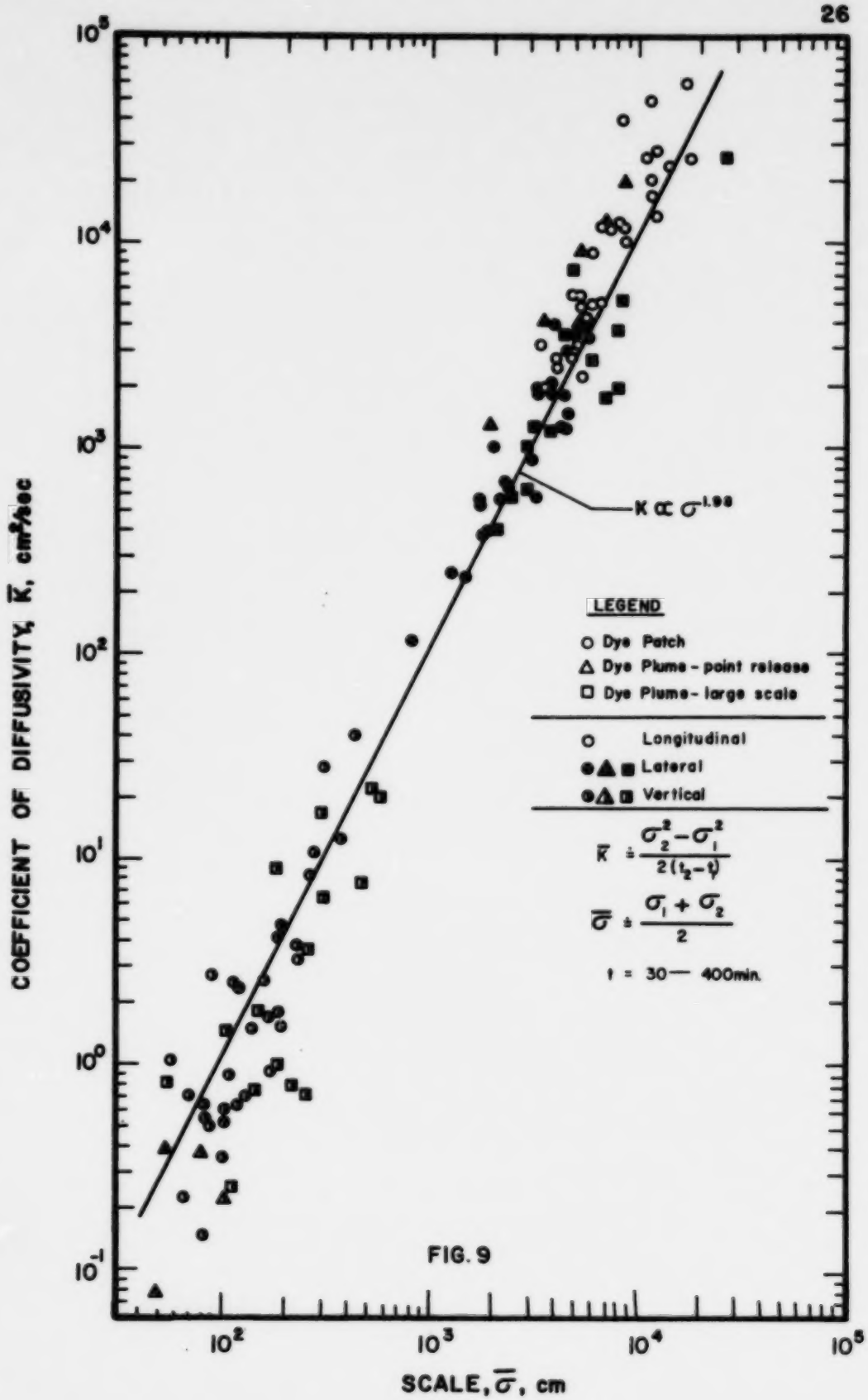

COEFFICIENT OF DIFFUSIVITY, $\bar{K}$, cm²/sec
SCALE, $\bar{\sigma}$, cm
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^2
10^3
10^4
10^5
$K \propto \sigma^{1.98}$
LEGEND
Dye Patch
Dye Plume - point release
Dye Plume - large scale
Longitudinal
Lateral
Vertical
$\bar{K} \doteq \frac{\sigma_2^2 - \sigma_1^2}{2(t_2 - t_1)}$
$\bar{\sigma} \doteq \frac{\sigma_1 + \sigma_2}{2}$
t = 30 — 400min.

FIG. 9

scatter of the plotted points in Figure 9 is attributed to differences in observation times. It is also evident that if the results of the present experiments were compared to those obtained by other investigators involving significantly different observation times, an unlimited number of empirical relationships would be possible. Such a comparison could easily result in an apparent corroboration of the 4/3 or, for that matter, any other power law.

Ultimate Coeficients. Orlob (1959), (1962) emphasized the fact that according to the similarity theory of homogeneous turbulence, the scale-diffusivity relationship should include a third variable, the rate of energy dissipation ϵ. Thus, for a variable ϵ, Orlob found that the ultimate (constant) lateral coefficient of diffusivity K_y, corresponding to the asymptotic phase of diffusion as observed in a small laboratory channel, could be expressed by

$$K_\infty \propto \epsilon^{1/3} L_a^{4/3} \tag{15}$$

where L_a is the Lagrangian mean eddy size. Ichiye (1960), in a discussion of Orlob's results, showed that according to energy spectrum theory the similarity principle can be expressed by

$$K_\infty \propto U^{-1} \epsilon^{2/3} L_\infty^{5/3} \tag{16}$$

where U is the mean current speed and L_∞ is the characteristic or ultimate eddy scale which was assumed by Orlob to be proportional to L_a.

In the present experiments the observed differences in the rates of horizontal diffusion may have been due to variable rates

of energy dissipation or shear conditions as related to daily changes in oceanographic conditions. As in most previously reported experiments on large-scale diffusion phenomena, it was not possible in the present experiments to determine appropriate values of either ϵ or L_{∞}. Consequently, direct verification of equation (15) or (16) was not possible. In many of these experiments, however, the lateral diffusivity coefficients were found to be nearly constant ($\sigma_y^2 \propto t$) and thus approximately equal to K_{∞}; therefore it seemed reasonable to assume that the initial observed values of σ_y were proportional to the ultimate eddy scale L_{∞}. Assuming minor variations in $\bar{U}$ or ϵ, a plot of $\bar{K}_y$ calculated from equation (7) vs. the initial observed value of σ_y might be expected to follow equation (15) or (16). Figure 10 is a log-log plot of these variables for both dye patch and large-scale plume experiments. Although there is considerable scatter of the plotted points, which is undoubtedly related to the assumptions stated above, the correlation coefficient for these data is 0.8582 which is significant at less than the 0.1% level. The results of the t-test indicated that the slope of the trend line is not different from 4/3 but is different from 5/3 or 2/1 at the 1% level of significance. These results, although crude, appear to agree with Orlob's experimental finds.

A similar relationship for the mean vertical and longitudinal coefficients could not be established from these data. As will be discussed later, $\bar{K}_z$ appeared to be related more to environmental conditions, e.g., water column stability and wind speed than to initial size.

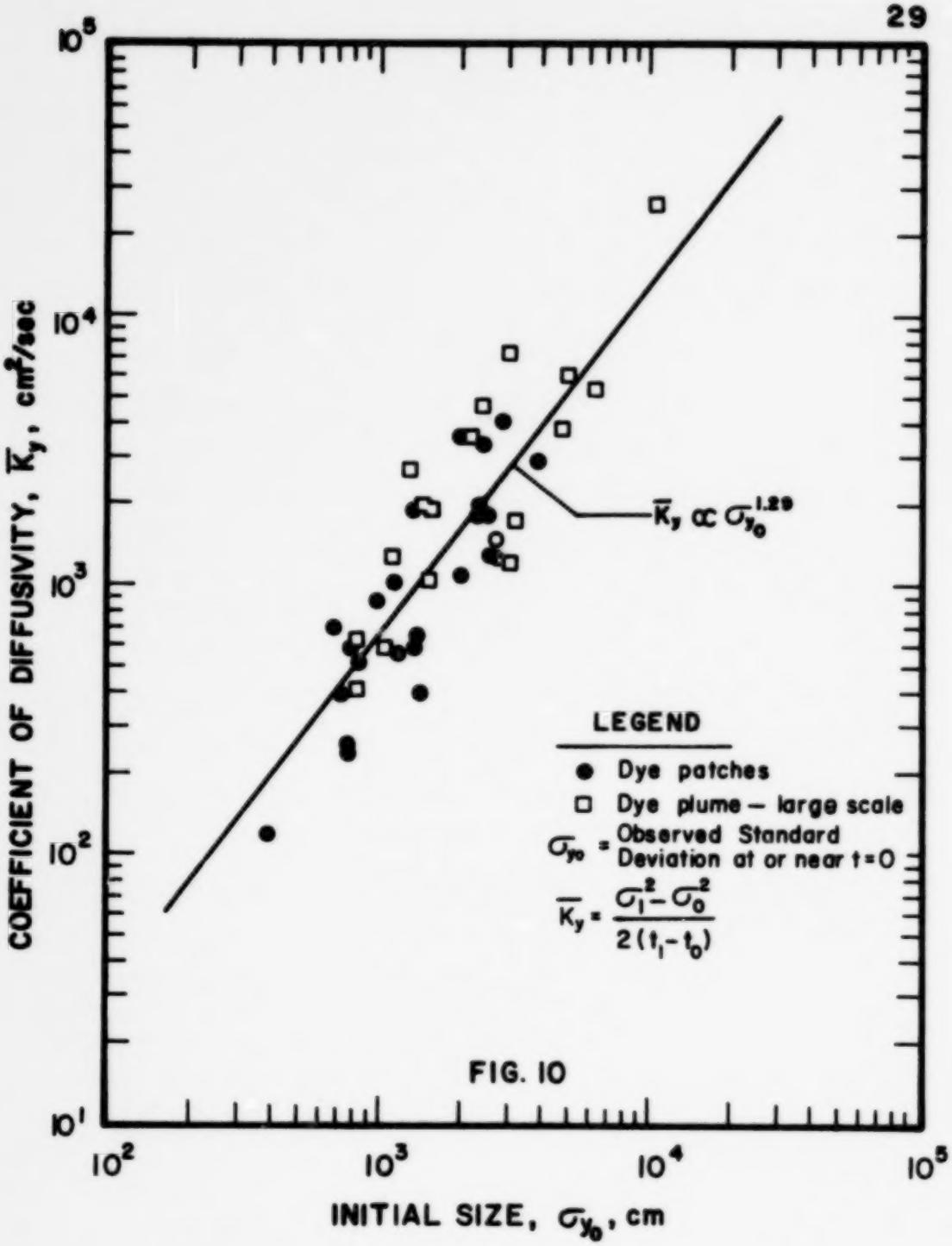

COEFFICIENT OF DIFFUSIVITY, $\bar{K}_y$, cm²/sec
INITIAL SIZE, σ_{y_0}, cm
10^5
10^4
10^3
10^2
10^1
10^2
10^3
10^4
10^5
$\bar{K}_y \propto \sigma_{y_0}^{1.29}$
LEGEND
Dye patches
Dye plume – large scale
σ_{y_0} = Observed Standard Deviation at or near t=0
$\bar{K}_y = \frac{\sigma_1^2 - \sigma_0^2}{2(t_1 - t_0)}$

FIG. 10

Effect of Environmental Factors

General. One of the principal objectives of these experiments was to assess the effects of various physical characteristics of the marine environment on turbulent diffusion. Previous attempts to establish correlations between vertical and horizontal rates of growth of dye patches in the ocean and such environmental factors as wind speed, water column stability and current speed were presented by the Allan Hancock Foundation (1965) and Foxworthy (1965). The data presented by these authors appeared to show that the rate of increase of the vertical variance σ_z^2 of dye patches was related to mean wind speed and water column stability but not the mean surface current speed. The rate of vertical growth was observed to be greatest in those experiments with relatively high wind speeds and low stabilities. These authors also suggested that in the nearshore coastal areas off southern California the stability in the surface layers is itself strongly influenced by wind action. Higher mean wind speeds generally resulted in lower mean stabilities and hence increased vertical diffusion of dye tracer. Therefore it was proposed that vertical diffusion near the water surface in the immediate coastal areas was controlled primarily by wind induced turbulence. Unfortunately the data presented were too few to substantiate this hypothesis.

Concerning the horizontal diffusion of dye patches, the above authors found little evidence to substantiate the fact that the rate of horizontal growth of dye patches and continuous dye plumes was influenced significantly by the environmental factors listed above.

These results agree, at least qualitatively, with previous concepts of diffusion in the ocean. It has long been speculated that

a stable water column (increasing density with depth) tends to suppress vertical turbulence. On the other hand, it has commonly been assumed that variations in water column stability per se have no affect on horizontal diffusion, that is, the horizontal spread of material is believed to proceed along planes of equal density.

It was hoped that the data from the experiments described herein, together with that presented earlier by the Allan Hancock Foundation would lead to a more thorough understanding of the interrelationship between oceanic diffusion and certain relevant environmental factors.

Density Considerations. Measurement of the vertical component of diffusion is complicated by the fact that in density stratified waters the rate of vertical transport of material may vary with depth. The present experiments involved the diffusion of dye within the relatively shallow surface layers (0-10m) of the ocean. For this reason diffusivity coefficients as computed from observed values of σ_z^2 and equations (1) and (7) were considered representative of average values for this depth interval.

Another phenomenon which may preclude the accurate measurement of vertical diffusion in stratified waters is the action of internal waves. Although such waves are not believed to contribute to the net vertical transport of diffusing material, their presence interferes with sampling techniques designed to measure true vertical diffusion. A recent study by Cairns and LaFond (1966) off the coast of southern California showed the existance of predictable vertical oscillations of the depth of temperature discontinuity (thermocline). They found that vertical oscillations of the thermocline were always present and composed of both short- and long-period components. The

short-period oscillations were characterized by wave heights of 1.7m and periods of 7.3 minutes. The longer-period oscillations, based on hourly averages of thermocline depth were found to be predictable and mainly a function of wind and tidal action. The magnitude of these vertical oscillations was found to be strongly correlated with mean hourly wind speed, exhibiting a diurnal cycle which places the thermocline for a given season at its least depth at approximately 0600 hrs., increasing to its maximum depth at about 1800 hrs.

Similar phenomena were observed in many of the present dye patch and plume experiments. In those studies in which the tracer was released at or near the water surface, the depth of dye concentration remained well above the thermocline and consistantly increased with diffusion time. Plots of observed values of σ^2 vs. time (cf. Figures 1 through 3) for the vertical spread of dye were very similar to those for the horizontal spread presumedly uninfluenced by internal wave motion. Therefore, it was assumed in these particular experiments that the influence of internal waves on vertical diffusion was minimal.

In some of the larger-scale experiments, however, the initial depth of the tagged waste plume above the outfall extended from the surface to the thermocline. Under such conditions the observed σ_z^2-t relationships showed a highly irregular pattern and in some cases σ_z^2 actually appeared to decrease with time. These results were assumed to be due to upward vertical oscillations of internal waves or possible buoyancy effects.

The discussion which follows pertains only to the results of the experiments in which the action of internal waves was considered

negligible. In this case, the observed rates of horizontal and vertical growth of dye patches and plumes was assumed to be controlled primarily by the interaction of wind speed, water column stability and possible current speed.

In these studies the water column stability was computed as the slope of the density-depth curve as defined by the equation

$$S = \frac{d\sigma_t}{dz} \doteq \frac{\Delta\sigma_t}{\Delta z} \qquad (17)$$

where $\sigma_t = (\rho - 1)\ 10^3$, ρ is the density at the in situ temperature and salinity and atmosphere pressure, and z is the depth in centimeters.

The data presented in Figure 11 depicts the observed vertical density structure for two typical dye patch experiments. These data were obtained near the center of a dye patch as it drifted with the prevailing current. The data from the experiment conducted within the waste field formed over the Orange County outfall shows the density structure as a function of distance from the origin of the plume or "boil" to a distance downstream of over a thousand meters. It is evident that in both experiments values of S computed from equation (17) were variable depending on the time of observation and depth interval used. For purposes of comparison the surface and 300 cm depth were arbitrarily selected as the significant depth interval for calculations of S. The weighted mean stability, $\overline{S}$, was computed for the entire time of the experiment and considered representative of the average conditions for any particular experiment.

The data shown in Figure 11 also demonstrates the entirely

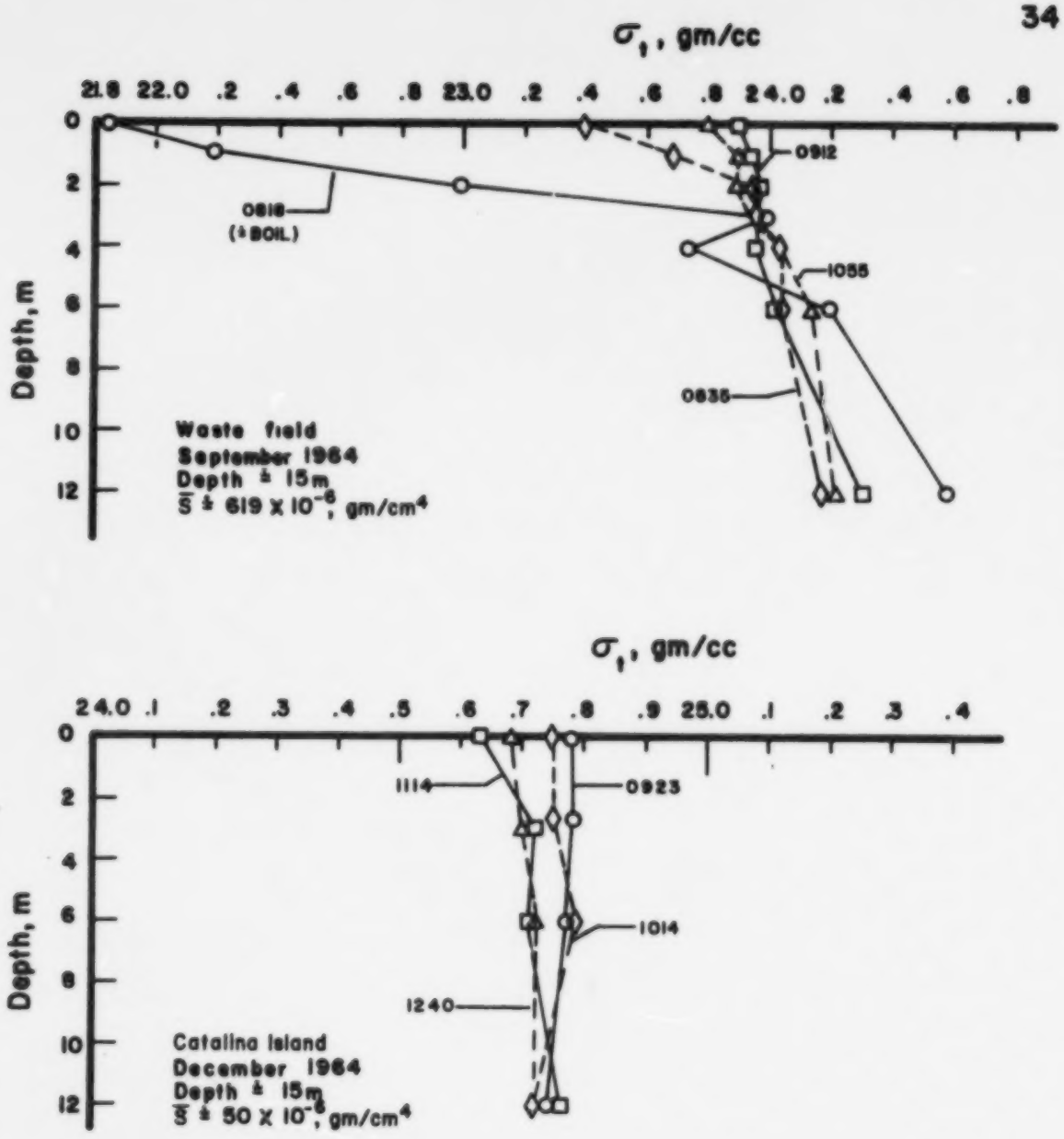

σt, gm/cc
21.8 22.0 .2 .4 .6 .8 23.0 .2 .4 .6 .8 24.0 .2 .4 .6 .8
0
2
4
6
8
10
12
Depth, m
0912
0818
(½ BOIL)
1055
0835
Waste field
September 1964
Depth ± 15m
S̄ ± 619 X 10⁻⁶, gm/cm⁴
σt, gm/cc
24.0 .1 .2 .3 .4 .5 .6 .7 .8 .9 25.0 .1 .2 .3 .4
0
2
4
6
8
10
12
Depth, m
1114
0923
1014
1240
Catalina Island
December 1964
Depth ± 15m
S̄ ± 50 X 10⁻⁶, gm/cm⁴

FIG. 11

different density conditions usually observed within surface waste fields and in the much deeper background waters near Catalina Island. The water column within the waste field was highly stratified (stable) as compared to that observed in this particular background area. The mean stabilities for dye patch experiments conducted in near-shore (mainland) coastal background areas were similar to those observed within waste fields dispersing in the same general areas. The lowest values of $\overline{S}$ were consistently observed in the waters near Catalina Island.

Figure 12 is a semi-log plot of mean stability against mean wind speed for several dye patch and plume experiments. These data show a strong negative correlation between the variables for the experiments conducted inside and outside surface waste fields in the near-shore coastal areas. The data collected off Catalina Island are insufficient to establish any relationship between the variables. It appears, however, that at the same mean wind speed the mean stabilities in the near-shore waters are approximately an order of magnitude higher than observed off Catalina Island.

These results appear to substantiate the previous hypothesis that the mean stability near the water surface in the near-shore waters off southern California is a function of mean wind speed.

Vertical Diffusivity. The relationship between mean stability and mean vertical diffusivity coefficients as computed from equation (7) is shown in Figure 13. The grouped data from the mainland coastal waters show a strong negative correlation between these variables when plotted on log-log paper. The data collected off Catalina Island, although few, appear to show a similar trend; however, these data points are displaced considerably to the left of the

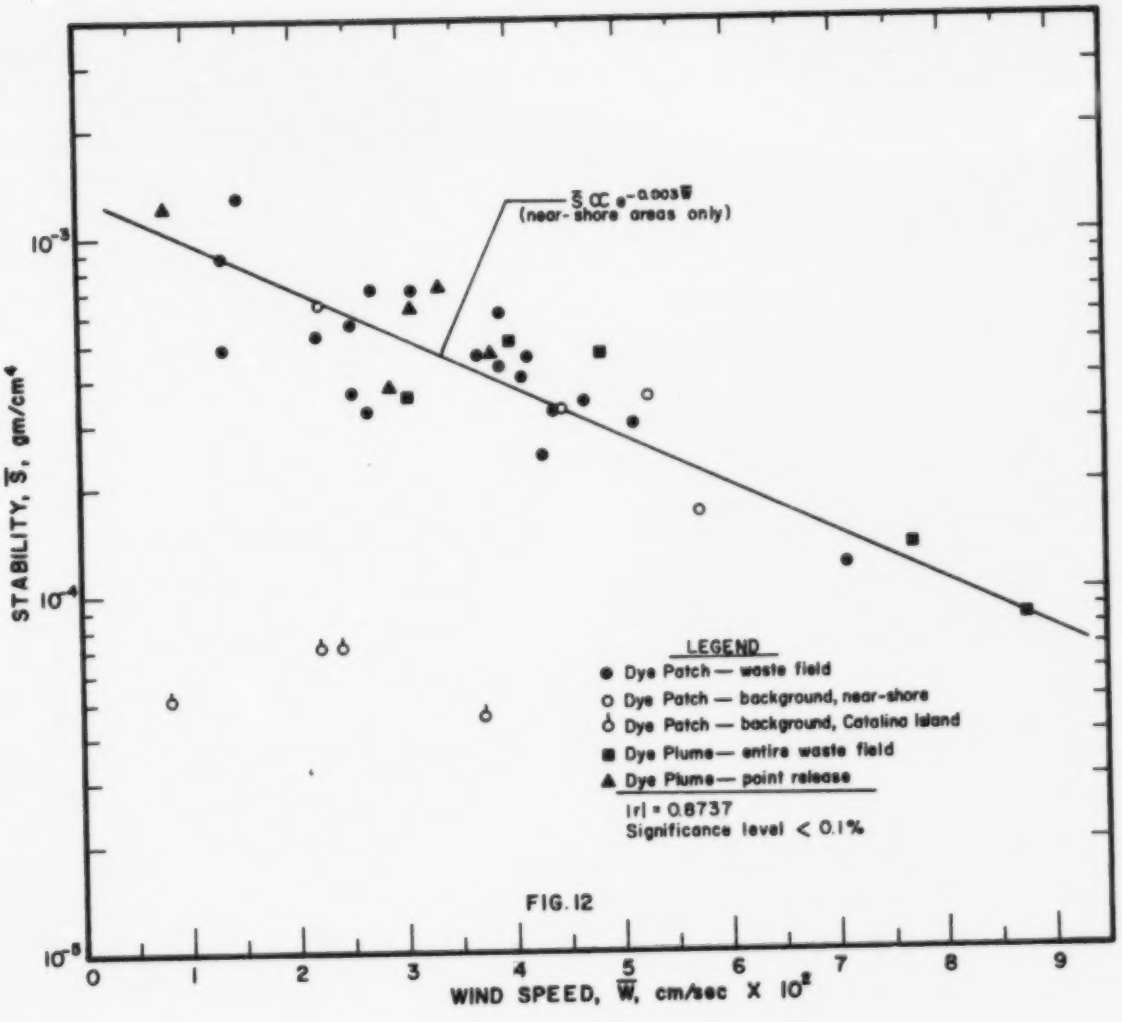

$\bar{S} \propto e^{-0.003\bar{W}}$
(near-shore areas only)
STABILITY, $\bar{S}$, gm/cm⁴
10^{-3}
10^{-4}
10^{-5}
LEGEND
Dye Patch — waste field
Dye Patch — background, near-shore
Dye Patch — background, Catalina Island
Dye Plume — entire waste field
Dye Plume — point release
|r| = 0.8737
Significance level < 0.1%
0
1
2
3
4
5
6
7
8
9
WIND SPEED, $\bar{W}$, cm/sec X 10^2

FIG. 12

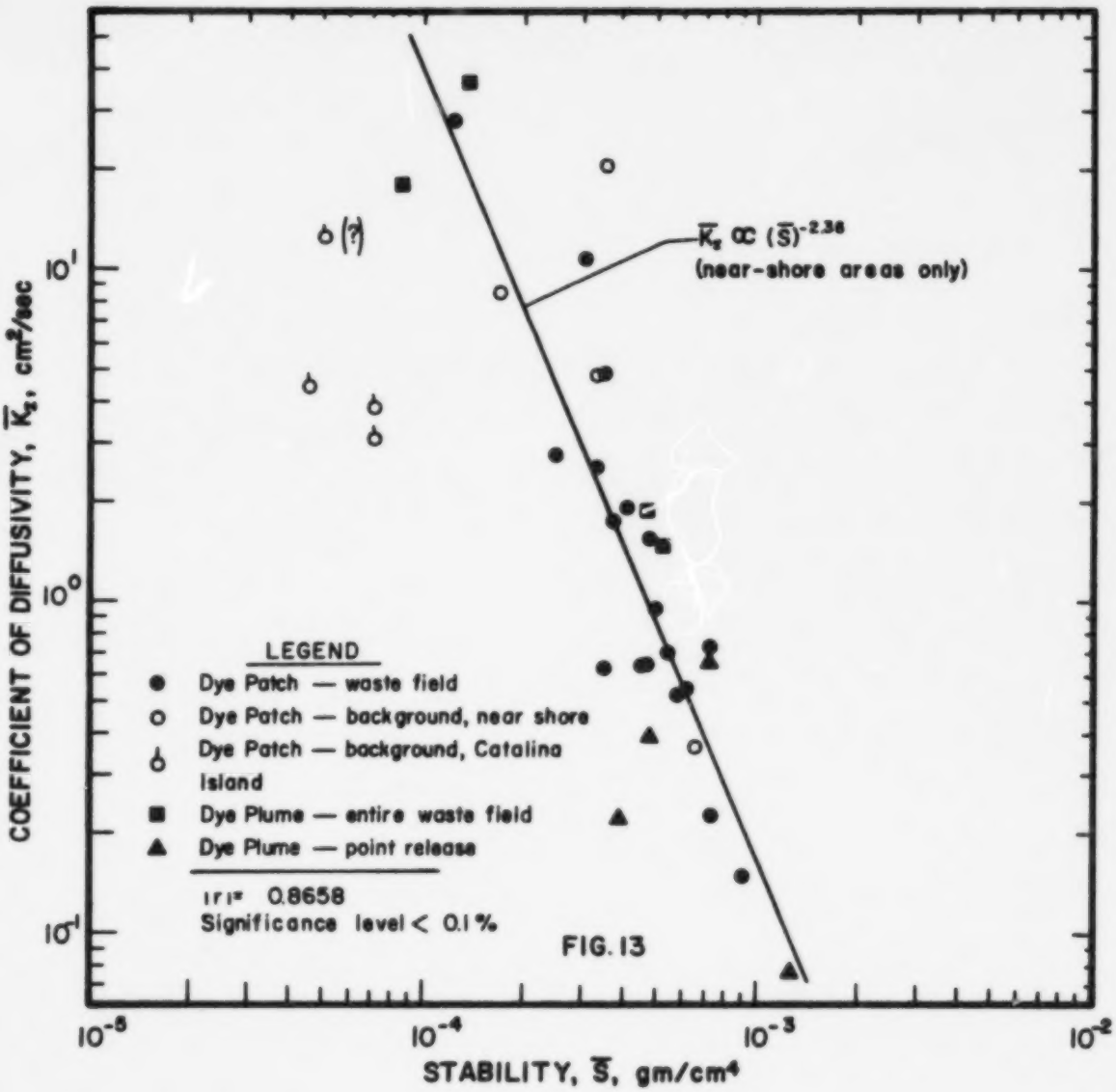

$\bar{K}_z \propto (\bar{S})^{-2.38}$
(near-shore areas only)
LEGEND
Dye Patch — waste field
Dye Patch — background, near shore
Dye Patch — background, Catalina Island
Dye Plume — entire waste field
Dye Plume — point release
|r|* 0.8658
Significance level < 0.1 %
COEFFICIENT OF DIFFUSIVITY, $\bar{K}_z$, cm²/sec
STABILITY, $\bar{S}$, gm/cm⁴
10^{1}
10^{0}
10^{-1}
10^{-5}
10^{-4}
10^{-3}
10^{-2}

FIG. 13

trend line for the near-shore areas. These data appear to show that the same value of $\overline{K}_z$ may result under considerably different stability conditions as related to particular oceanic areas.

It is apparent from the data presented in Figures 12 and 13 that $\overline{K}_z$ is related to mean wind speed. This relationship is shown graphically in Figure 14. These data indicate that there is a strong trend for $\overline{K}_z$ to increase with increasing wind speed particularly in the near-shore areas.

Although the results are not shown here, no significant correlation could be established between $\overline{K}_z$ and mean surface current speed $\overline{U}$. This result was rather surprizing since one would expect $\overline{U}$ to be strongly related to $\overline{W}$ and hence related to $\overline{K}_z$. The fact that this was not the case in these experiments is emphasized by the lack of significant correlation between $\overline{W}$ and $\overline{U}$ as shown in Figure 15. These results do not agree with those presented by Stevensen (1958). He found that in the near-shore area off Newport Beach, California the mean current speed was approximately 2% to 3% of the mean wind speed. The reason for the apparent discrepancy in experimental results may be due to the methods used to measure surface current speeds. Stevensen employed drift cards in most of his experiments. Even though the cards were designed to minimize the influence of wind drag, it is doubtful that this method of velocity measurement is as accurate as the methods employed in the present experiments.

The results of the analyses discussed above appear to show that vertical diffusion near the water surface in the oceanic areas studied is more universally related to mean wind speed than any other single observed environmental factor. This conclusion is

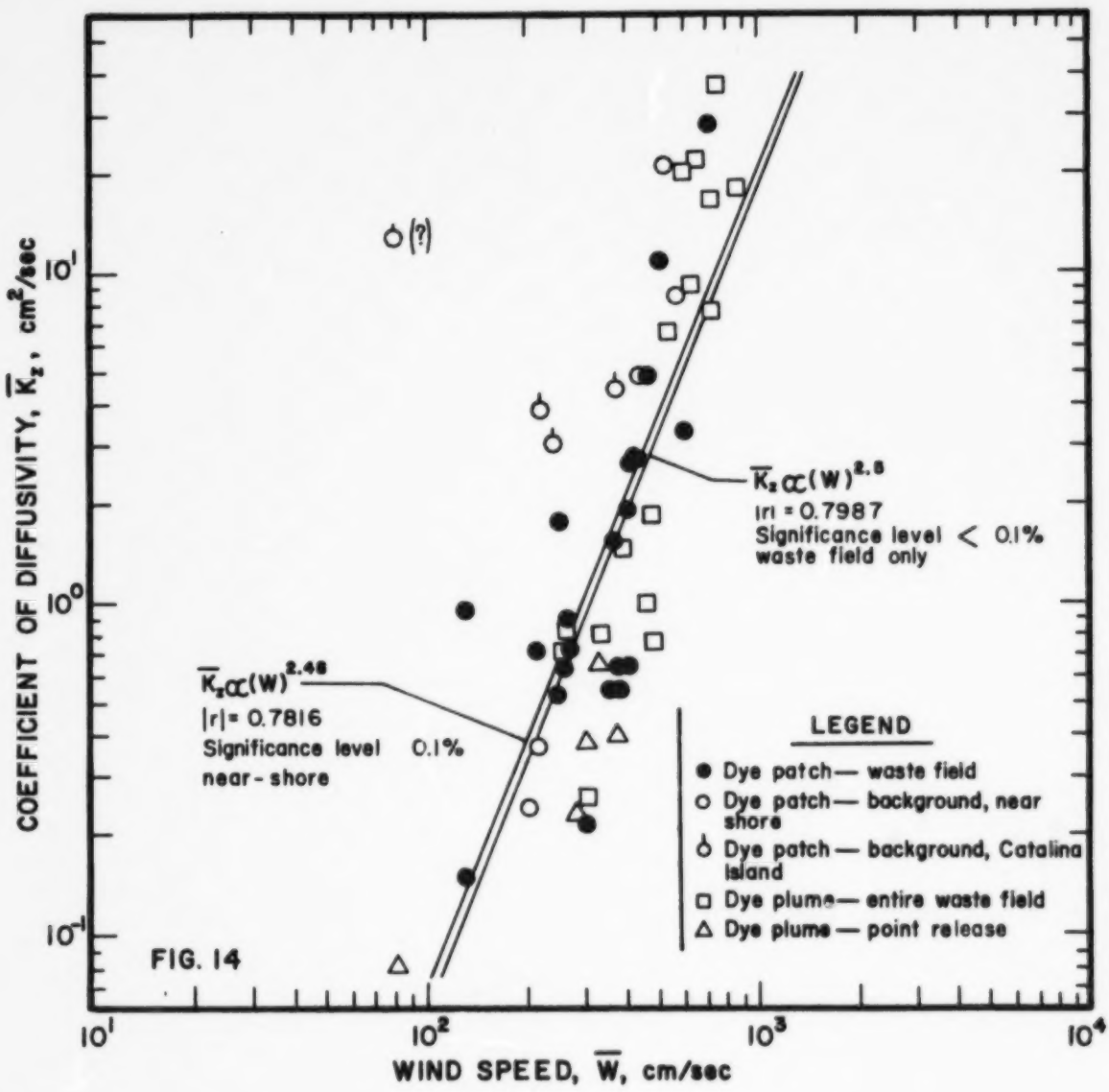

COEFFICIENT OF DIFFUSIVITY, $\bar{K}_z$, cm²/sec
WIND SPEED, $\bar{W}$, cm/sec
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
$\bar{K}_z \propto (W)^{2.8}$
|r| = 0.7987
Significance level < 0.1%
waste field only
$\bar{K}_z \propto (W)^{2.48}$
|r| = 0.7816
Significance level 0.1%
near-shore
(?)
LEGEND
Dye patch— waste field
Dye patch— background, near shore
Dye patch — background, Catalina Island
Dye plume— entire waste field
Dye plume — point release

FIG. 14

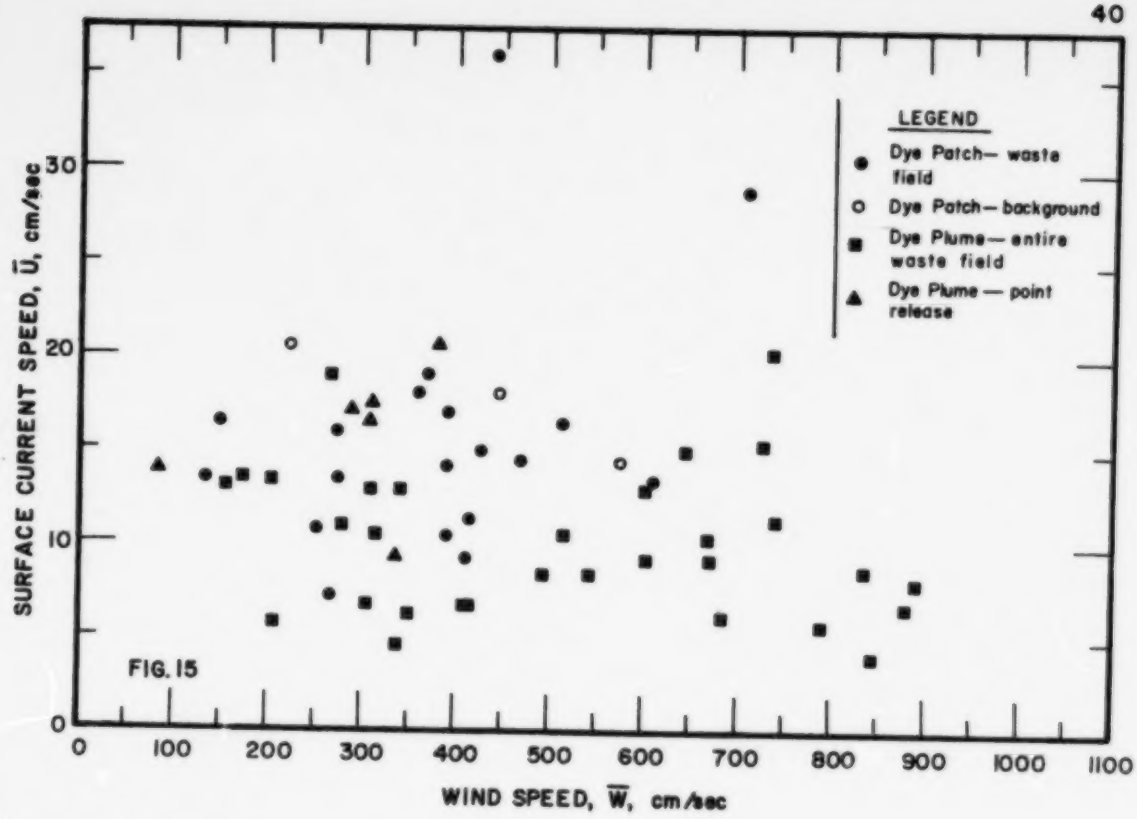

LEGEND
Dye Patch— waste field
Dye Patch— background
Dye Plume— entire waste field
Dye Plume — point release
SURFACE CURRENT SPEED, Ū, cm/sec
WIND SPEED, W̄, cm/sec
0
10
20
30
0
100
200
300
400
500
600
700
800
900
1000
1100

FIG. 15

obviously a tenuous one since the effect of other factors, e.g., Richardson's Number (see Okubo 1962,b) which involves vertical gradients of current speed and density, were not considered here. Unfortunately measurements of the vertical distribution of current speed in these experiments were not considered accurate enough to include Richardson's Number as a factor in these analyses. Regardless of this fact, $\overline{K}_z$ appears to be strongly related to mean wind speed as shown by the highly significant correlations between $\overline{K}_z$ and $\overline{W}$ shown in Figure 14.

It is interesting to compare the empirically derived relationships between $\overline{K}_z$ and $\overline{W}$ with previously proposed relationships between the vertical coefficient of eddy viscosity and wind speed. Based on the earlier work of Ekman (1902) and Thorade (1914), Sverdrup et al (1942) derived the following empirical relationships:

$$A_z \propto W^3 \; ; \; W < 600 \quad {}^{cm}/_{sec}$$

$$A_z \propto W^2 \; ; \; W > 600 \quad {}^{cm}/_{sec}$$

where A_z (g/cm/sec/) is the mean vertical coefficient of eddy viscosity "inthe surface layer stirred up by the wind". Although there is considerable scatter of the data shown in Figure 14, it is evident that Sverdrup's predictions relating A_z and W are similar to the observed relationships between $\overline{K}_z$ and $\overline{W}$.

Horizontal Diffusivity. True horizontal diffusion in the ocean is the result of purely random fluctuations of relatively small-scale turbulent eddies. Thus a dye patch released at the ocean surface will spread horizontally due to internal mixing promoted by random motion of eddies of a size equal to or smaller than the size of the patch. In this case the environmental factors affecting horizontal diffusion are those which influence horizontal turbulent

velocity fluctuations. In the absence of lateral boundaries, it might be speculated that wind speed, current speed and possibly water column stability would be important factors influencing horizontal diffusion in the ocean.

Recently the role of large-scale eddies in the overall dispersion process has become better understood. It is now known that the motion of relatively large-scale eddies result in non-uniformity in the mean current velocity. Non-uniform currents produce shears in both vertical and horizontal directions. Thus, the horizontal spread or dispersion of a dye patch may be brought about by the combined effect of random turbulent motion and current shear. The significance of the so-called "shear effect" along any horizontal axis of a dye patch depends on the orientation of the prevailing vertical velocity structure. In the case of non-skewed uniform currents, horizontal and vertical shear will cause a dye patch to elongate and mix in the direction of the mean current, i.e., in the so-called longitudinal direction. Recently Carter and Okubo (1965) and Okubo (1968) discussed the importance of the "shear effect" on horizontal diffusion. They proposed a series of theoretical models for the three-dimensional rates of growth of dye patches dispersing in a uniform current. According to these authors the coordinate variances are time dependent as given by the following equations:

$$\sigma_x^2 = \left(\frac{2}{3}\,\Omega_y^2\,Ky + \frac{1}{6}\,\Omega_z^2\,K_z\right) t^3 \qquad (18)$$

$$\sigma_y^2 = 2\,K_y t \qquad (19)$$

$$\sigma_z^2 = 2\,K_z t \qquad (20)$$

where σ_x^2, σ_y^2 and σ_z^2 are the coordinate variances in the longitudinal, lateral and vertical directions respectively, K_y

and K_z are the constant lateral and vertical diffusivity coefficients, and Ω_y and Ω_z are the associated coordinate shears. According to the above equations the "shear effect" is concentrated in the X or longitudinal direction of dispersing patch of material.

In the case of pure shear-diffusion the mean longitudinal diffusivity coefficient, $\bar{K}_x$, as computed from equations (1) and (18) related to $\bar{K}_y$, $\bar{K}_z$, Ω_y, and Ω_z. It appeared reasonable, therefore, to assume that the coordinate diffusivities and shears are in turn related to wind speed, current speed, and possibly water column stability, i.e., the same environmental factors assumed to affect diffusion due to purely random eddy motion.

As mentioned previously in most of the dye patch experiments the patch was observed to elongate markedly in the direction of the mean current. Investigations of the vertical current structure (see Foxworthy and Barsom (1967)) showed that in many of these experiments the current speed decreased with increasing depth with the subsurface currents varying only slightly in direction from the surface currents. Hence in these experiments it was assumed that a uniform current prevailed and the effect of current shear was concentrated in the longitudinal direction of the dispersing patches and plumes. For these conditions the equations proposed by Carter and Okubo (1965) for the various coordinate rates of growth were assumed to apply.

In Figures 16 through 18, values of $\bar{K}_y$ for all experiments are plotted against several different environmental factors. It is evident that little if any correlation exists between $\bar{K}_y$ and mean values of stability, wind speed, or current speed. In addition no correlation could be established between $\bar{K}_y$ and any combination

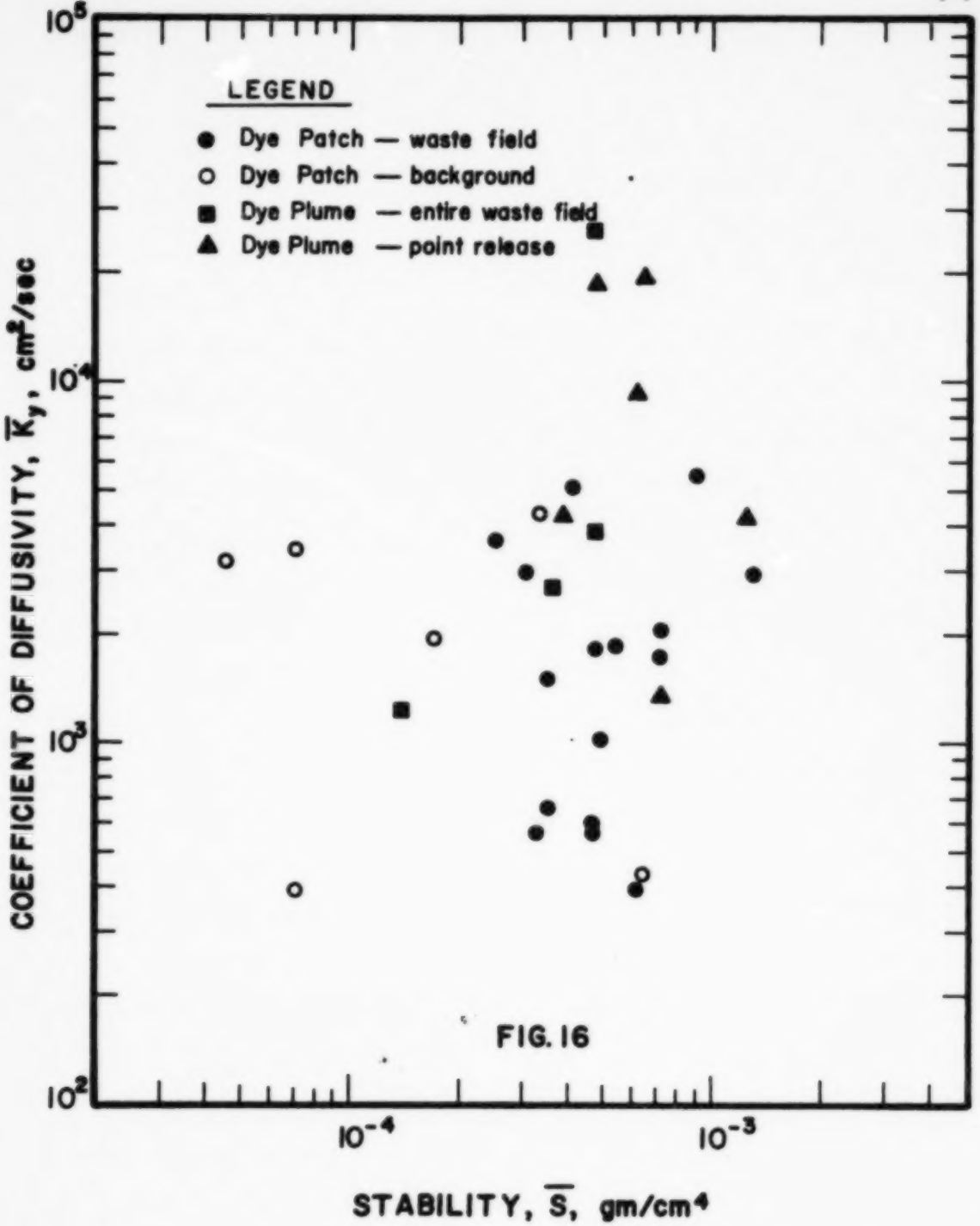

LEGEND
Dye Patch — waste field
Dye Patch — background
Dye Plume — entire waste field
Dye Plume — point release
COEFFICIENT OF DIFFUSIVITY, $\overline{K}_y$, cm²/sec
10^5
10^4
10^3
10^2
10^{-4}
10^{-3}
STABILITY, $\overline{S}$, gm/cm⁴

FIG. 16

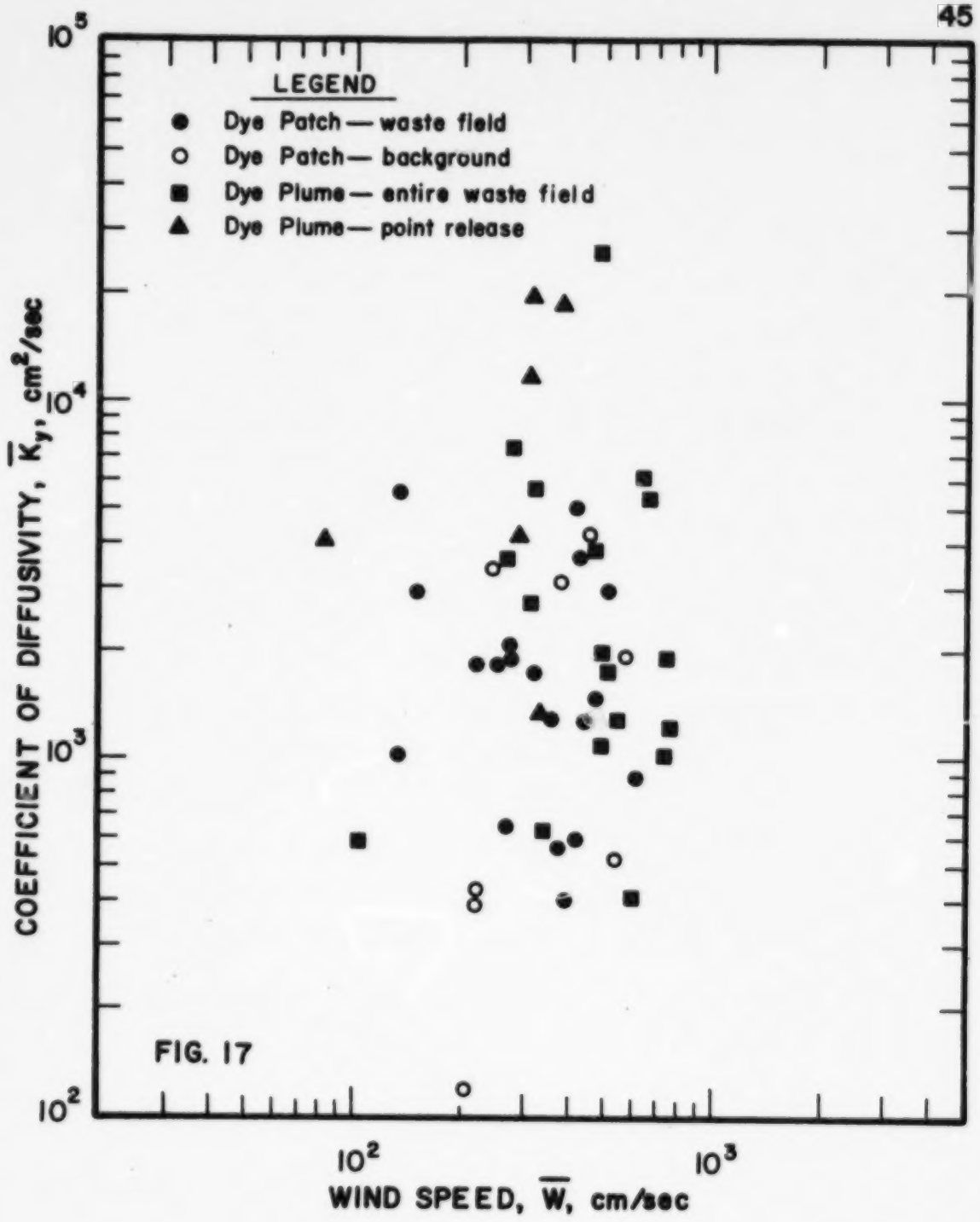

LEGEND
Dye Patch — waste field
Dye Patch — background
Dye Plume — entire waste field
Dye Plume — point release
COEFFICIENT OF DIFFUSIVITY, $\overline{K}_y$, cm²/sec
10^5
10^4
10^3
10^2
10^2
10^3
WIND SPEED, $\overline{W}$, cm/sec

FIG. 17

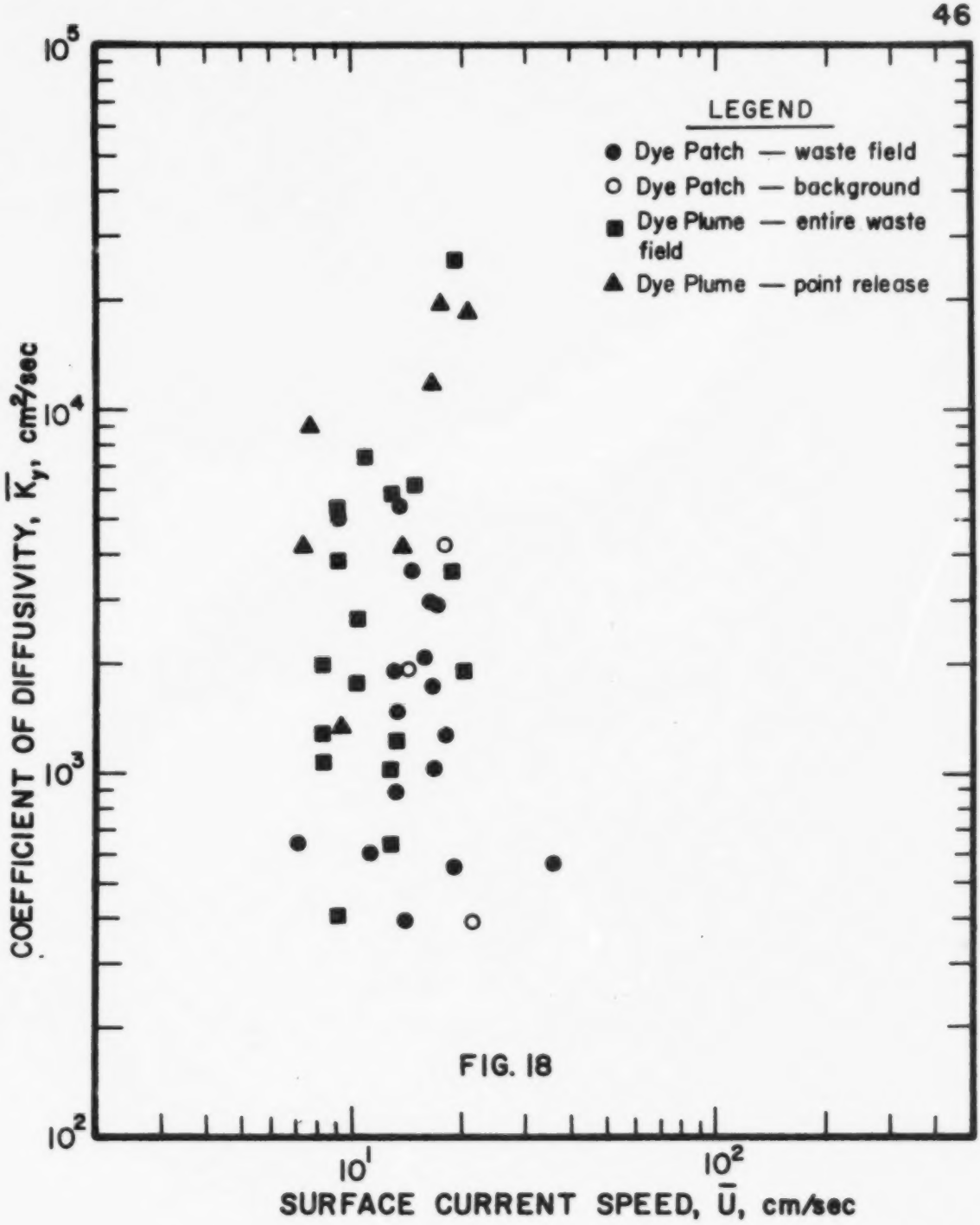

LEGEND
Dye Patch — waste field
Dye Patch — background
Dye Plume — entire waste field
Dye Plume — point release
COEFFICIENT OF DIFFUSIVITY, $\bar{K}_y$, cm²/sec
10^5
10^4
10^3
10^2
10^1
10^2
SURFACE CURRENT SPEED, $\bar{U}$, cm/sec

FIG. 18

of these factors. These results appear to show that $\bar{K}_y$ is related more to scale effect, as measured by σ_{yo} (cf. Figure (10)), than any single environmental factor studied.

It is also interesting to compare the results of the full-scale field experiments with those obtained by Masch (1964) in diffusion experiments conducted in a small laboratory channel. Masch found that the lateral coefficients observed in these experiments were constant and related to the surface current speed and "average wave age" defined as the ratio of wave speed to wind speed. In Masch's experiments there was a strong general trend for K_y to increase with increasing current speeds. A similar trend is not evidenced by the data presented in Figure 18. This result may be due to inaccuracies involved in the measurement of $\bar{U}$, or to the fact that the data presented in Figure 18 represent a much wider range of "wave age" than observed by Masch. In this regard it is interesting to note that in the full-scale plume experiments there was a strong tendency for the mean initial width of the waste plumes to decrease with increasing current speed. In particular the ratio of the initial width of the field, as measured by σ_{yo}, to the length, Y, of submarine diffusers was strongly correlated with current speed. This relationship is depicted in Figure 19 and shows that for a particular outfall higher currents are associated with smaller initial field widths. Since σ_{yo} was shown previously to be strongly correlated with $\bar{K}_y$ (cf. Figure 10)), one would expect $\bar{K}_y$ to be related to $\bar{U}$ as demonstrated by Masch.

The relationship between the longitudinal diffusivity coefficients $\bar{K}_x$ and several environmental factors are shown in Figures 20 through 22. These data are for dye patch experiments only since $\bar{K}_x$ cannot

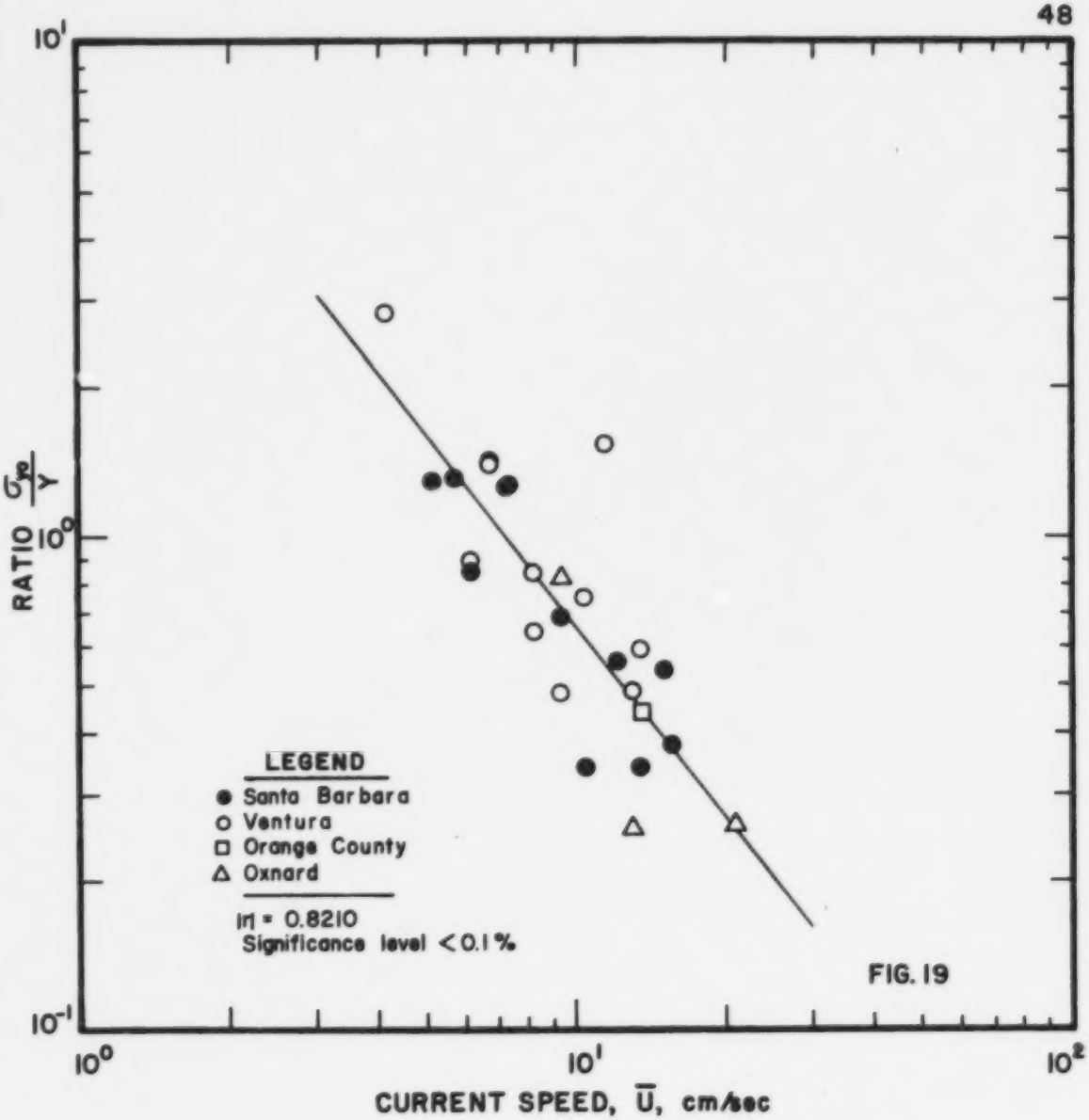

RATIO $\frac{\sigma_e}{Y}$
10^1
10^0
10^{-1}
10^0
10^1
10^2
CURRENT SPEED, $\bar{U}$, cm/sec
LEGEND
Santa Barbara
Ventura
Orange County
Oxnard
$|r| = 0.8210$
Significance level < 0.1 %

FIG. 19

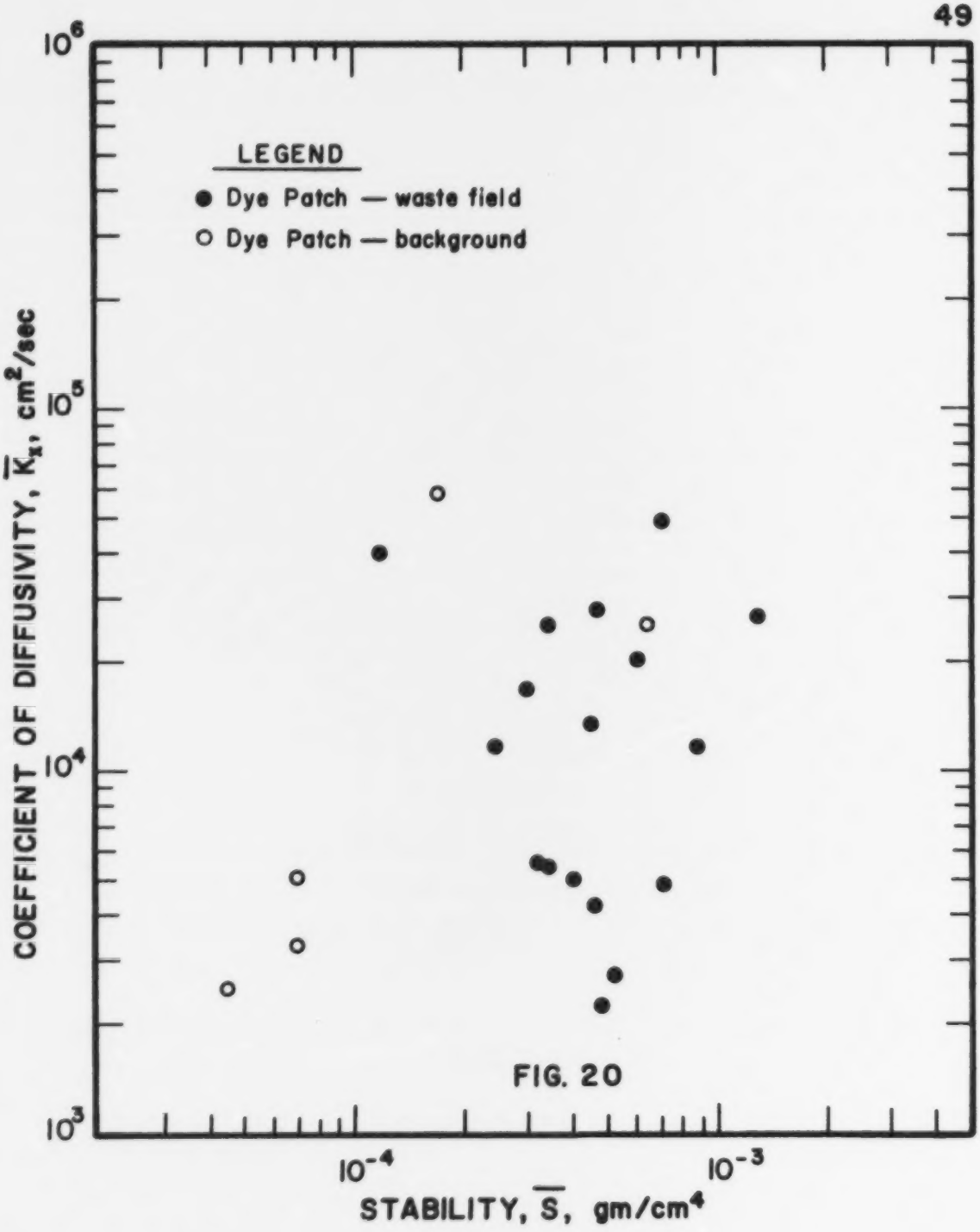

LEGEND
● Dye Patch — waste field
○ Dye Patch — background
COEFFICIENT OF DIFFUSIVITY, $\bar{K}_x$, cm²/sec
10^6
10^5
10^4
10^3
10^{-4}
10^{-3}
STABILITY, $\bar{S}$, gm/cm⁴

FIG. 20

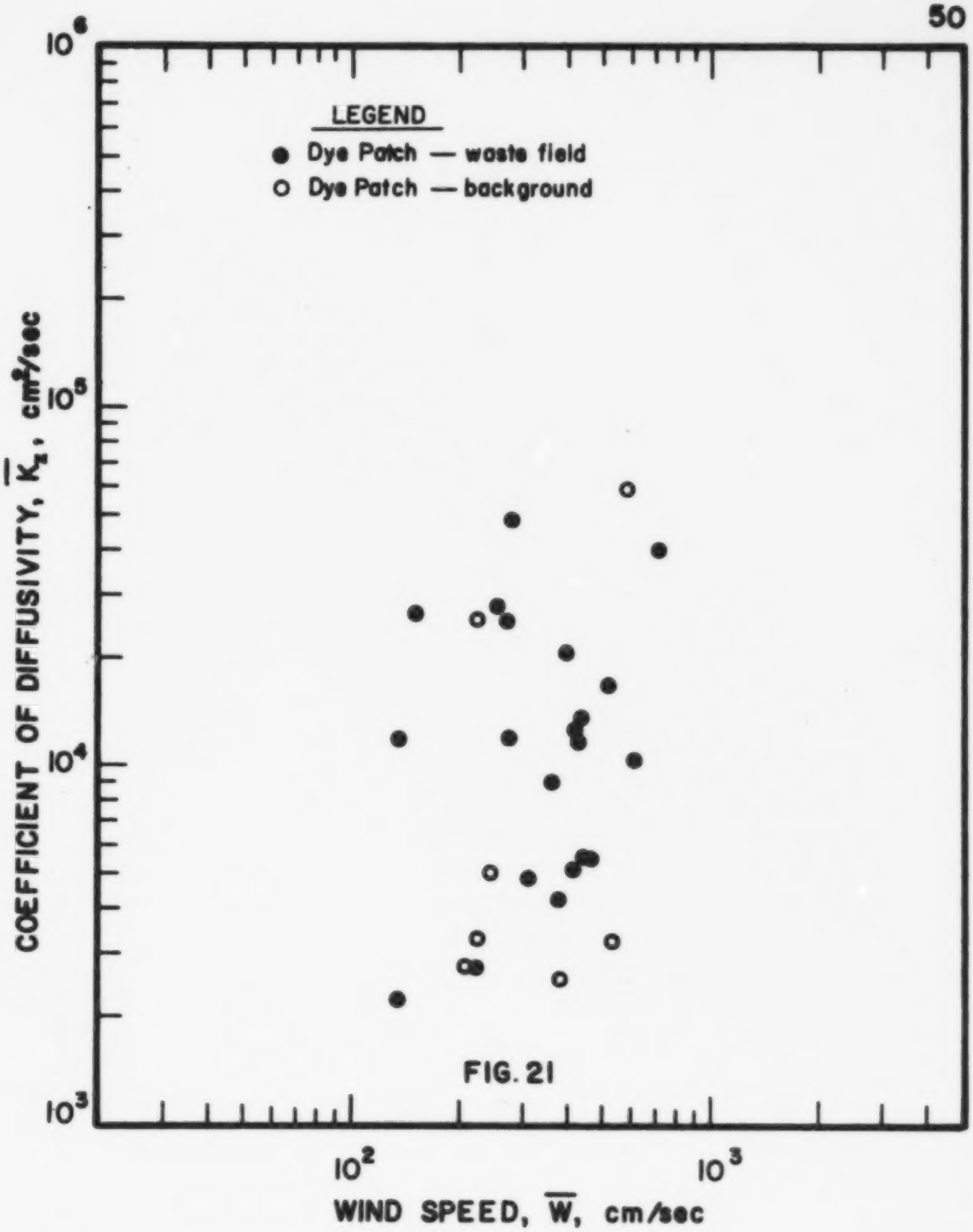

LEGEND
Dye Patch — waste field
Dye Patch — background
COEFFICIENT OF DIFFUSIVITY, $\overline{K_x}$, cm²/sec
10^6
10^5
10^4
10^3
10^2
10^3
WIND SPEED, $\overline{W}$, cm/sec

FIG. 21

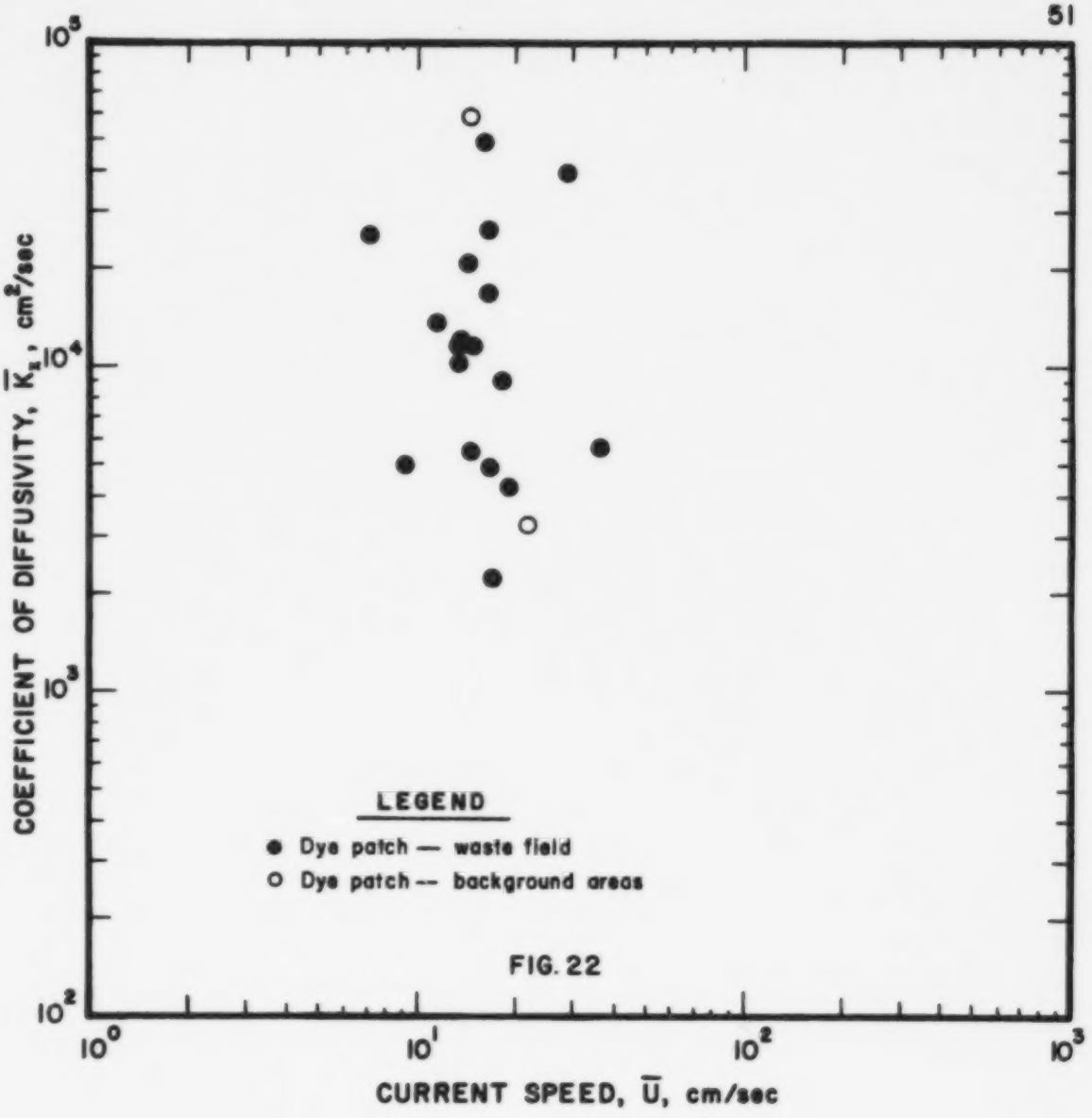

COEFFICIENT OF DIFFUSIVITY, $\bar{K}_x$, cm²/sec
10^5
10^4
10^3
10^2
LEGEND
Dye patch — waste field
Dye patch -- background areas
10^0
10^1
10^2
10^3
CURRENT SPEED, $\bar{U}$, cm/sec

FIG. 22

be determined in continuous plume experiments. As in the case of lateral diffusivity coefficients, no correlation could be found between $\bar{K}_x$ and stability, wind speed, or current speed. According to equation (18) σ_x^2 (and therefore $\bar{K}_x$) is a function of K_z and K_y; therefore an attempt was made to establish a simple relationship between K_x and the other coordinate diffusivities. Examples of the results of this analysis are shown in Figures 23 and 24. It is evident that little or no correlation exists between these variables. In addition, no correlation could be found between $\bar{K}_x$ and any logical combination of the variables plotted in Figures 20 through 24.

The above results do not necessarily disprove the validity of equation (18), but rather reflect the fact that the "shear effect" cannot be described by simple relationships involving the environmental factors observed in these experiments. A more detailed investigation of vertical velocity structures existing during these experiments should have been made. However, it is important to note that for the case of constant shears and diffusivities, equation (18) predicts a $\sigma_x^2 \propto t^3$ relationship. As discussed in the first part of this report (cf. Figures (1) and (2)), no t^3 relationship was observed in any of the dye patch experiments.

It should also be emphasized that contrary to the findings for $\bar{K}_y$, no significant correlation could be established between $\bar{K}_x$ and the initial observed length of the dye patches as measured by σ_{xo}. In summary, $\bar{K}_x$ did not appear to be related to any single environmental or other factor investigated.

Practical Application of Results

The experiments described in this paper were also conducted

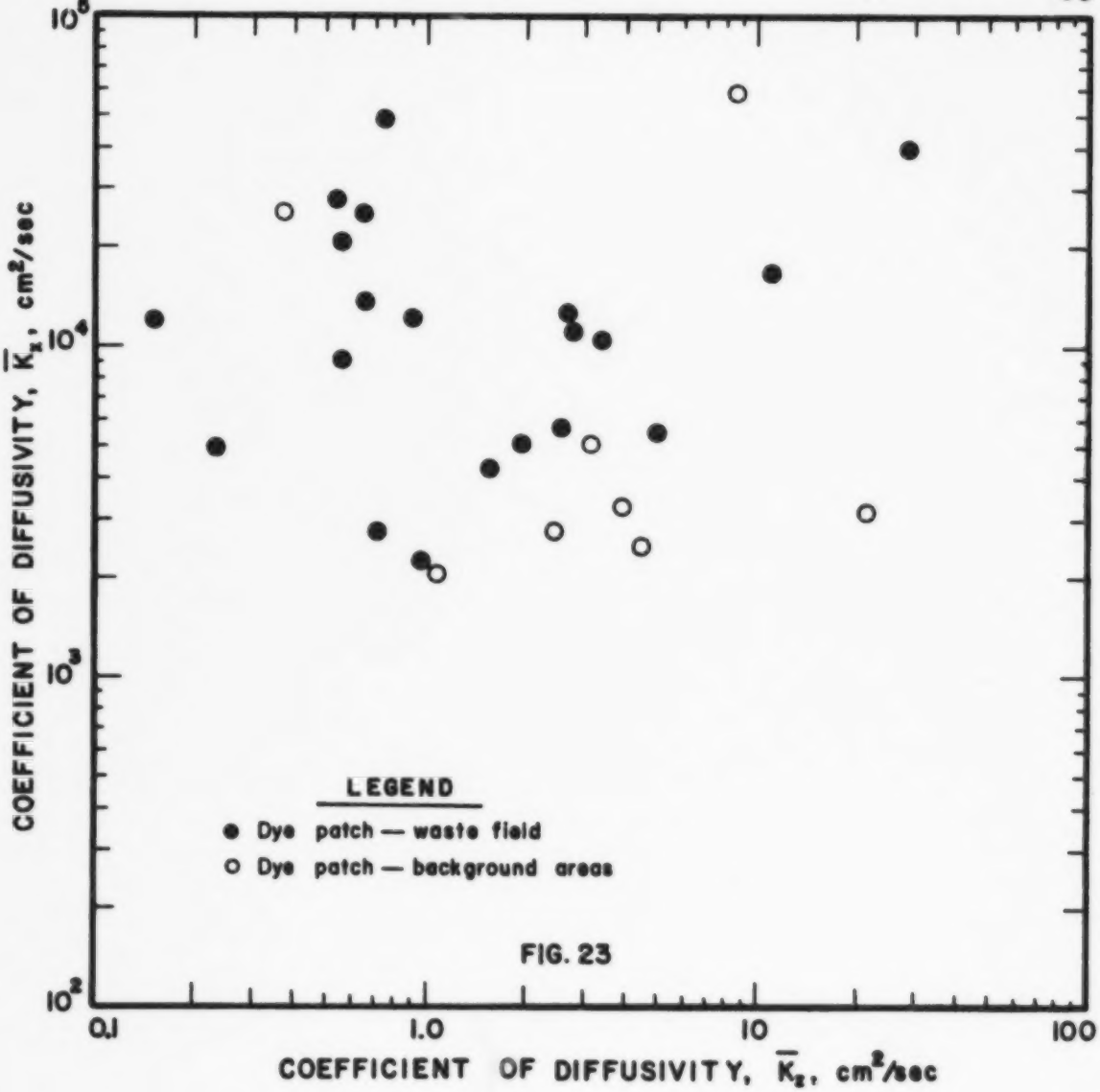

10^5
10^4
10^3
10^2
COEFFICIENT OF DIFFUSIVITY, $\bar{K}_x$, cm²/sec
LEGEND
● Dye patch — waste field
○ Dye patch — background areas
0.1
1.0
10
100
COEFFICIENT OF DIFFUSIVITY, $\bar{K}_z$, cm²/sec

FIG. 23

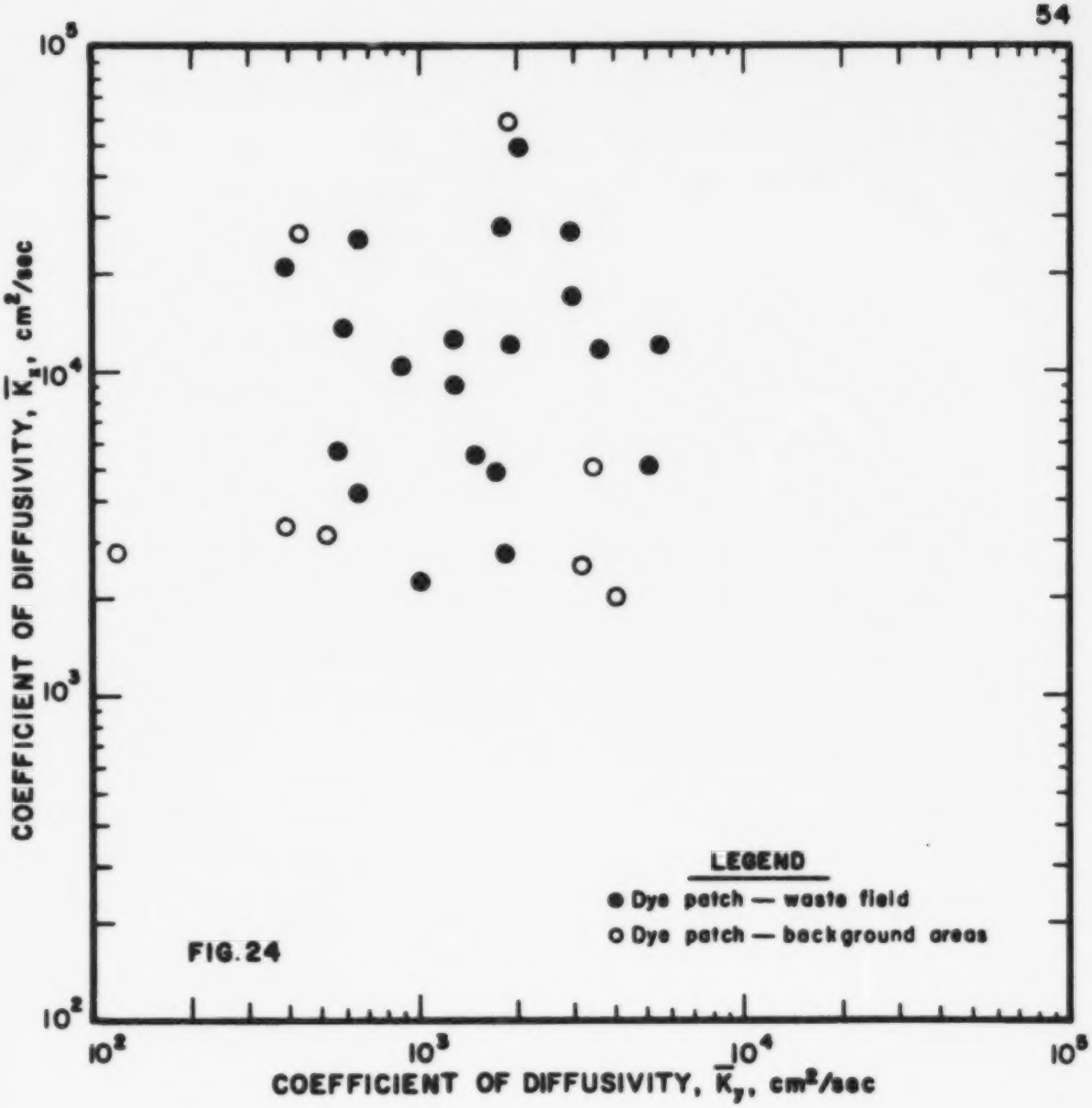

10^5
10^4
10^3
10^2
COEFFICIENT OF DIFFUSIVITY, $\bar{K}_x$, cm²/sec
LEGEND
● Dye patch — waste field
○ Dye patch — background areas
10^2
10^3
10^4
10^5
COEFFICIENT OF DIFFUSIVITY, $\bar{K}_y$, cm²/sec

FIG. 24

with a view toward obtaining data and defining parameters that would prove useful in the practical design of marine outfall systems. Technically speaking these results apply only to the particular oceanic areas studied and under rather limited environmental conditions. It is believed, however, that certain of these data possess sufficient generality to be useful, at least as supplemental information, in the design of waste outfalls located in near-shore coastal waters.

When designing ocean outfalls for the continuous disposal of an unstable waste material, the engineer must consider two distinct phenomena. The first is the physical dilution of the waste and ocean water which results in a decrease of waste concentration. The second phenomenon involves the natural decay or disappearance of waste dye to environmental and other factors. In this paper, only the effect of physical dilution will be considered; the combined effect of dilution and decay is the subject of another series of experiments now in progress.

The overall phenomenon of physical dilution is usually considered to occur in two phases. The first is the so-called initial dilution phase produced by the interaction between a particular diffuser system and local oceanographic conditions. The end of this phase is reached when the waste-ocean water mixture rises to a depth of density equilibrium and begins to move primarily under the influence of the prevailing current. Dilution of the waste plume as it moves with the current is the second phase and dilution occuring during this phase is called subsequent dilution. After discharge the net dilution at any point along the waste plume is the product of initial and subsequent dilutions.

The design engineer has considerable control over initial dilution and little or none over subsequent dilution. The initial dilution can be maximized by the design of an efficient diffuser system following the methods proposed by Rawn, Bowerman and Brooks (1960) and Harremoes (1968) among others. The problem here lies in the fact that it is usually economically or physically impossible to produce sufficient initial dilution to lower the waste concentration to an acceptable level. It is necessary, therefore, to rely on subsequent dilution (and decay for an unstable waste) to reduce the waste concentration to within allowable limits. It is in this regard that the data presented in this paper may prove useful.

As an illustration, consider the results obtained from two typical large-scale experiments on the city of Ventura outfall. It will be shown that for these surface plumes only the observed oceanographic conditions, outfall characteristics, and data presented herein are necessary to make an accurate estimate of the rate of subsequent dilution. The equations used in this analysis will be those presented by Foxworthy and Kneeling (1968) for one- and two-dimensional dispersion of a steady continuous large-scale surface plume. For the two-dimensional case with constant diffusivity coefficients these authors propose

$$C/C_o = \frac{1}{\left[\frac{2\,K_y t}{\sigma_{yo}^2} + 1\right]^{\frac{1}{2}} \left[\frac{2\,K_z t}{\sigma_{zo}^2} + 1\right]^{\frac{1}{2}}} \qquad (21)$$

where:

C/C_o = the ratio of maximum centerline concentration along the plume to the maximum concentration at the origin.

σ_0^2 = the lateral and vertical variances of tracer concentration distribution at the origin.

t = drift time from origin.

K = diffusivity coefficients.

For a steady plume with negligable vertical diffusion equation (21) reduces to

$$C/C_o = \frac{1}{\left[\frac{2 K_y t}{\sigma_{yo}^2} + 1\right]^{\frac{1}{2}}} \qquad (22)$$

Both of the above equations can be expressed in terms of centerline distance, x, along the plume by simply substituting $x/\bar{U}$ for t.

The following average conditions and outfall characteristics were observed during the experiment of August 2, 1967:

Flow rate, Q = 9.65×10^4 cm^3/sec
Diffuser length, = 1700 cm
Minimum Initial dilution, s., = 49/1
Wind speed, W = 840 cm/sec
Current speed, U = 4.1 cm/sec

From these data it is possible to estimate appropriate values of K and σ^2 for use in the solution of equations (21) and (22). From Figure 19 with a current speed of 4.1 cm/sec we obtain $\sigma_{yo}/Y = 2.1$, hence $\sigma_{yo} = 3570$ cm. With this value of σ_{yo} and Figure 10 we find the lateral diffusity coefficient to be $\bar{K}_y \doteq 3400$ cm^2/sec. The vertical coefficient can be estimated from Figure 14 using a wind speed of 840 cm/sec, the result is $\bar{K}_z \doteq 14 cm^2$/sec. The value of σ_{zo} can be estimated from the following simple continuity relationships:

$$\sigma_{yo}\ \sigma_{zo} \propto \frac{sQ}{U}$$

where s is the average initial dilution of waste at the origin of the surface plume. Since the minimum initial dilution s_o is proportional to s, the above relationship can be expressed as

$$\sigma_{yo}\,\sigma_{zo} = k\,\frac{s_o Q}{U} \qquad (23)$$

where k is a constant of proportionality. The relationship between these variables for the four outfalls studied is shown in Figure 25. Although there is considerable scatter of the plotted points, the correlation coefficient for the best fit trend line is highly significant. It is also apparent that the calculated slope of the trend line is not significantly different from the theoretical value of 1.0. Much of the scatter is attributed to difficulties involved in making accurate measurements of the vertical distribution of dye tracer at the origin of the surface plume over the outfall.

From the data given above the ratio $s_o Q/U$ is 1.15×10^5 cm^2. With this value and Figure 25 we obtain $\sigma_{yo}\,\sigma_{zo} = 5.9 \times 10^5$ cm^2 from which $\sigma_{zo} = 157$ cm. It is now possible to solve equations (21) and (22) for the estimated decrease in maximum waste concentrations along the centerline of the plume. The results are shown graphically in Figure 26 together with the best fit trend line for the observed maximum dye concentration distribution. It is evident that for the given conditions the two-dimensional equation (21) yields an estimate of the decrease in maximum dye concentration which is within acceptable engineering limits. On the other hand equation (22), which assumes negligable vertical diffusion, yields a relatively poor estimate of the observed conditions.

It is also interesting to compare the results of an experiment

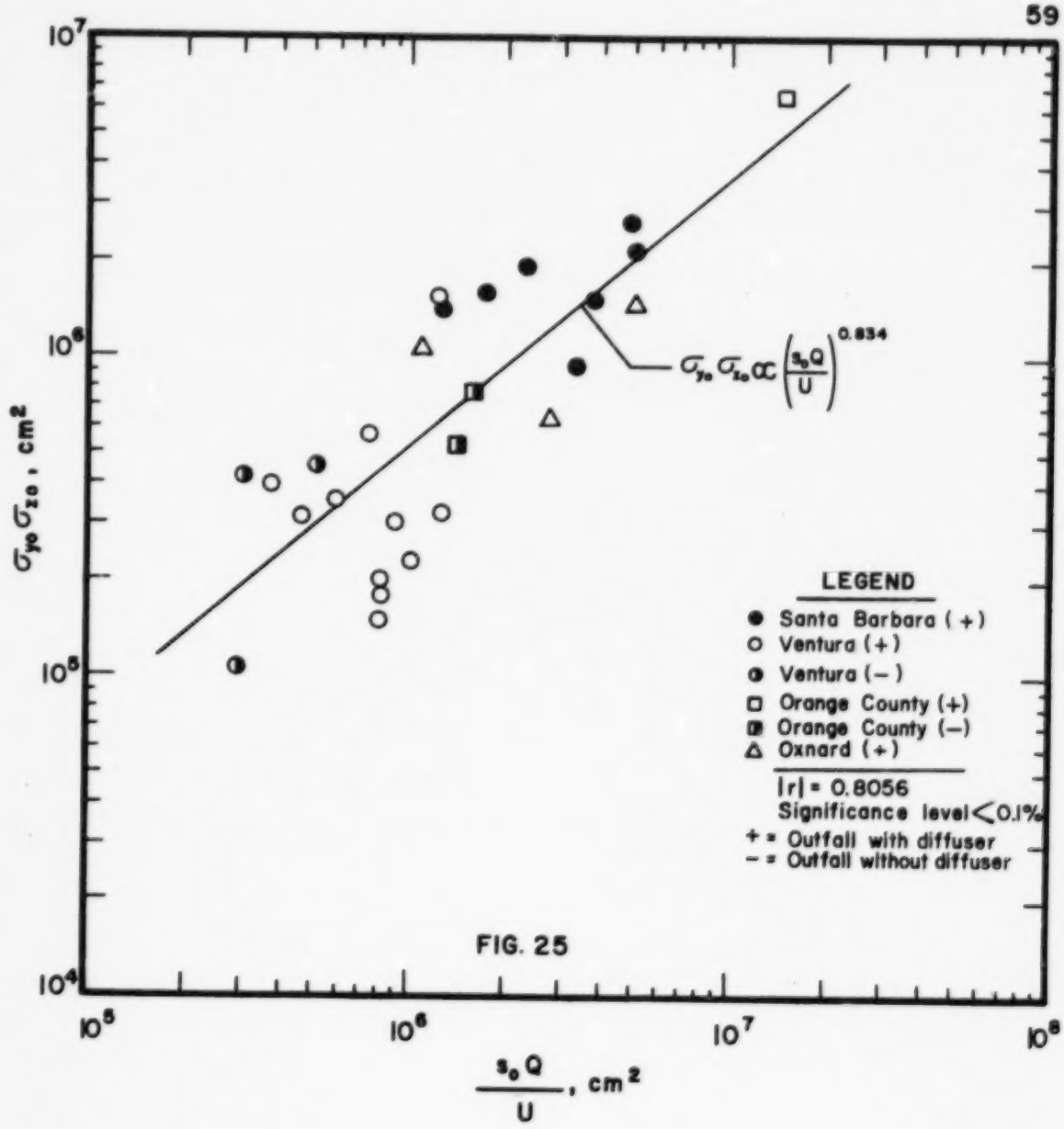

10^7
10^6
10^5
10^4
$\sigma_{y_0}\sigma_{z_0}$, cm²
$\sigma_{y_0}\sigma_{z_0} \propto \left(\frac{s_0 Q}{U}\right)^{0.834}$
LEGEND
Santa Barbara (+)
Ventura (+)
Ventura (−)
Orange County (+)
Orange County (−)
Oxnard (+)
|r| = 0.8056
Significance level < 0.1%
+ = Outfall with diffuser
− = Outfall without diffuser
10^5
10^6
10^7
10^8
$\frac{s_0 Q}{U}$, cm²

FIG. 25

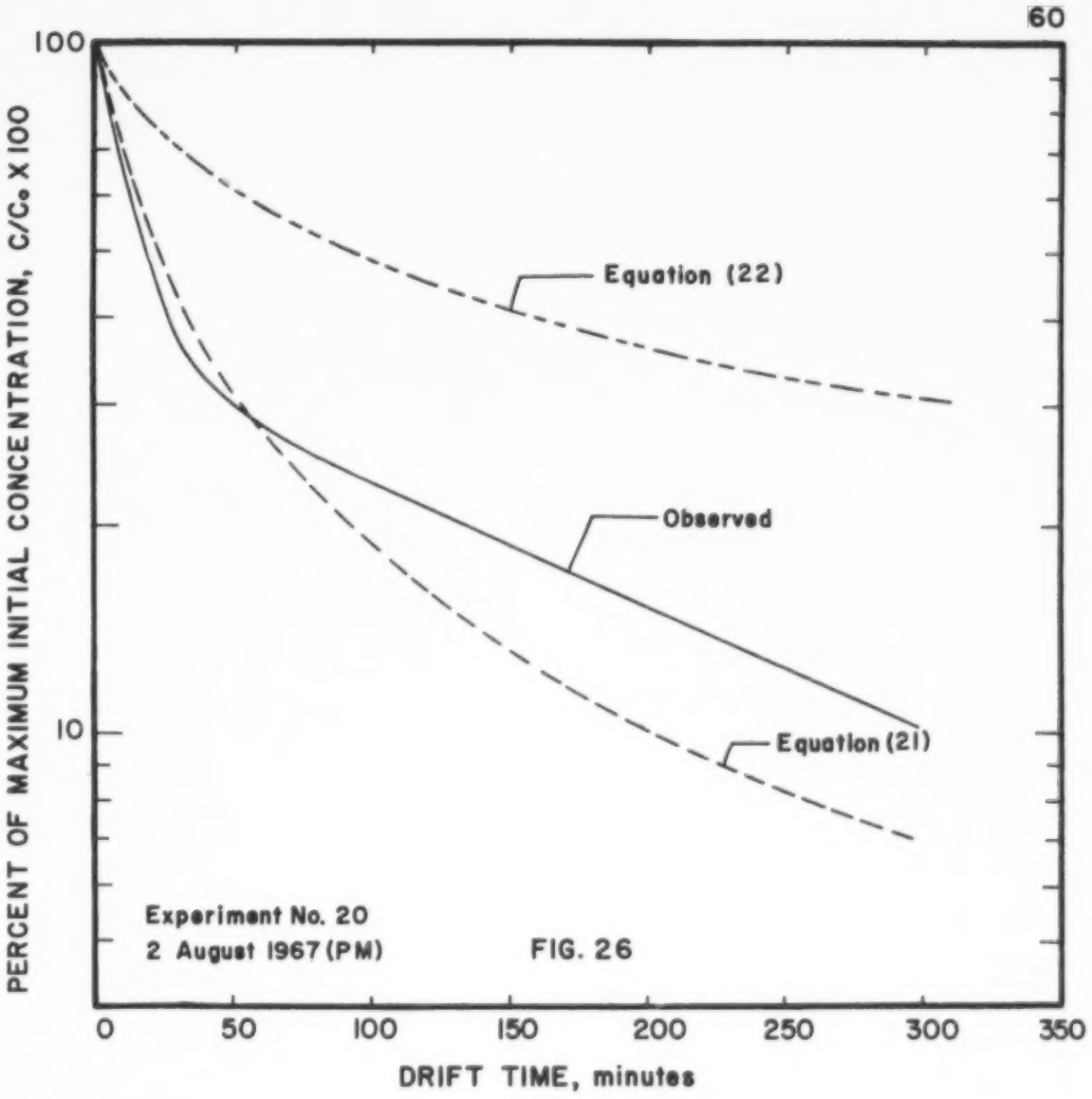

60
100
10
PERCENT OF MAXIMUM INITIAL CONCENTRATION, C/Co X 100
Equation (22)
Observed
Equation (21)
Experiment No. 20
2 August 1967 (PM)
0
50
100
150
200
250
300
350
DRIFT TIME, minutes

FIG. 26

in which vertical diffusion was minimal to that described above. The average test conditions for the experiment of February 24, 1968 are shown below.

$W = 203$ cm/sec
$U = 13.4$ cm/sec
$s_o = 99.5/1$
$Q = 1.1 \times 10^5$ cm^3/sec

Applying the same method for the solution of equations (21) and (22) yields the results shown in Figure 27. It is evident that the solutions of these equations are essentially identical. This indicated that vertical diffusion was negligible in this particular experiment. The agreement between the predicted and observed rates of decrease of dye concentration is good for the period of observation.

Conclusions

The data presented herein appear to substantiate the following conclusions:

1. For individual dye patch and continuous dye plumes the relationship between the instantaneous coefficients of diffusivity and approximate eddy scale did not follow the "four-thirds" law.
2. The instantaneous longitudinal and vertical diffusivity coefficients tended to be more nearly constant than time variable.
3. The mean diffusivity coefficients were found to vary with the approximate mean eddy scale raised to a power, the magnitude of which depends upon the time of observation.
4. The mean lateral diffusivity coefficients were found to be related to the approximate "initial size" of

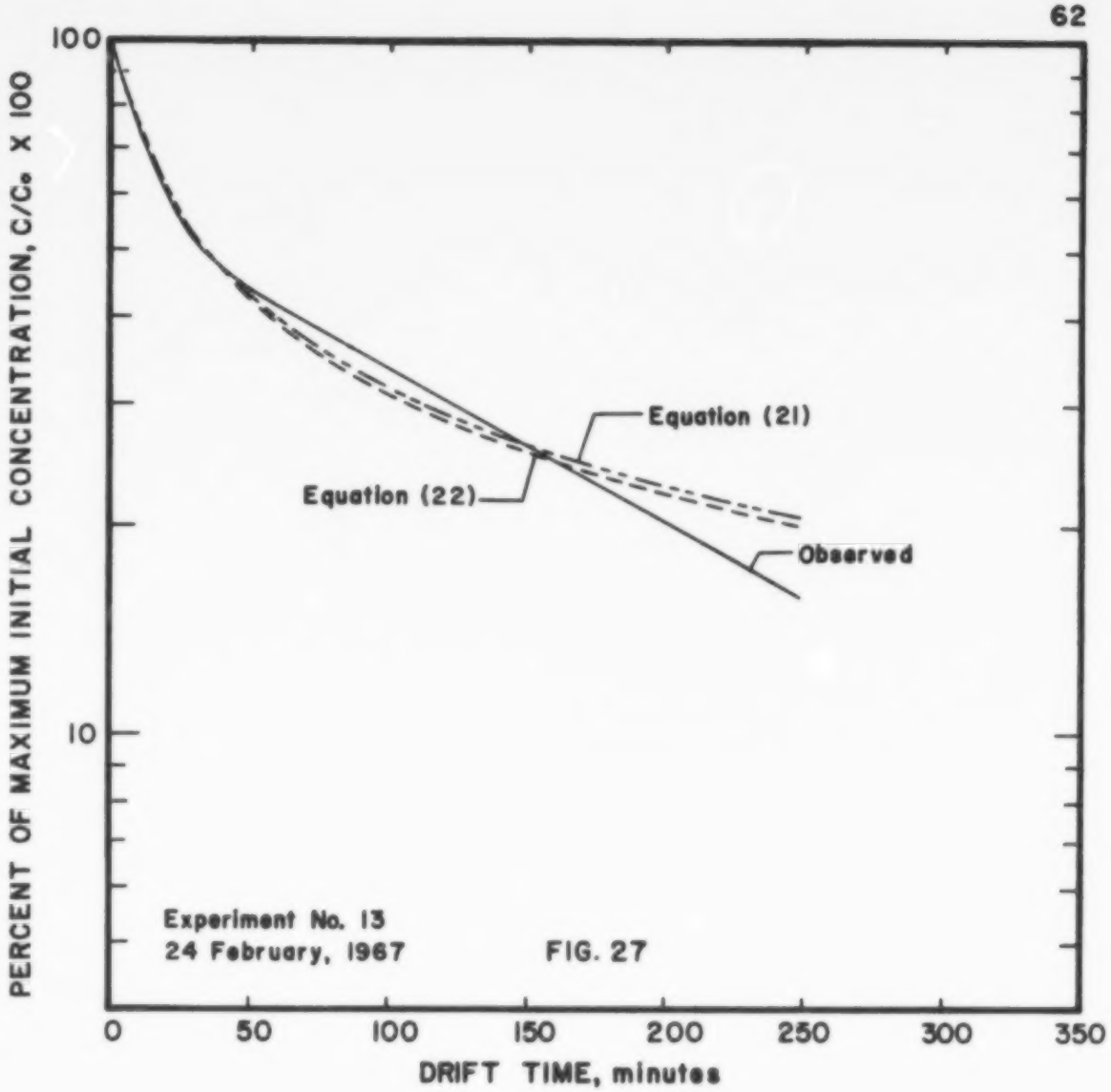

100
10
PERCENT OF MAXIMUM INITIAL CONCENTRATION, C/Co X 100
Equation (21)
Equation (22)
Observed
Experiment No. 13
24 February, 1967
0
50
100
150
200
250
300
350
DRIFT TIME, minutes

FIG. 27

dye patches and plumes. This relationship closely followed the "four-thirds" power law.

5. The mean vertical diffusivity coefficients were found to vary with water column stability as influenced by wind action near the water surface.
6. No relationship could be established relating the mean longitudinal diffusivity coefficients for dye patches and any single environmental factor studied.
7. The results of these experiments may be used to estimate the rate of subsequent dilution of continuous large-scale surface waste plumes dispersing in oceanic areas similar to those studied in this investigation.

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Table 1
Significance Tests (1)

Variance (2)	Degress of freedom, ν	Correlation Coefficient |r|	Significance level (%)
σ_x^2	3	0.997	0.1
σ_y^2	3	0.991	0.11
σ_z^2	2	0.991	1.0

Calculated values of |t| for indicated theoretical slope b_o			Tabulated values of |t| at 1% significance level
b_o= 1.0	b_o= 2.0	b_o= 3.0	5.841
7.57	5.26	18.1	5.841
5.50	1.93	9.36	9.925
0.88	8.89	18.65	

(1) Hoel, P. G., Introduction to Mathematical Statistics, International Textbook Co., 1962 Chapters 15 and 16.

(2) x = longitudinal, y = lateral, z = vertical coordinate directions.

(3) If |t| calculated is greater than |t| tabulated, it is concluded that the observed slope is different from the theoretical slope and the significance level indicates the probable error of this conclusion.

Table 2
Summary of
Oceanographic Conditions

No.	Date and Location		$\bar{W}$ (cm/sec)	$\bar{U}$ (cm/sec)	$\bar{S}$ (gm/cm^4) x 10^{-6}	$\bar{Q}$ (cm^3/sec)
I Dye Patch Experiments:						
2	10-31-62	O.C.	426	14.9	245	N.A.
3	11-1-62		220	--	530	
4	11-13-62		410	9.2	405	
5	11-14-62		133	13.3	880	
6	11-28-62		390	10.3	435	
7	11-29-62		251	10.7	365	
8	11-30-62		267	7.2	345	
10	3-12-63		415	11.3	460	
11	3-13-63		272	13.4	---	
12	3-14-63		610	13.3	---	
13	4-9-63		467	14.4	345	
14	4-11-63		---	13.8	---	
15	5-29-63		418	--	---	
16	5-29-63		359	18.0	---	
17	2-7-64		513	16.4	300	
18	2-9-64		149	16.4	1290	
19	9-25-64		390	14.1	610	
20	9-26-64am		133	16.9	490	
21	9-26-64pm		272	15.9	715	
22	12-15-64		369	19.0	465	
23	12-17-64		251	--	570	
24	4-15-65		708	28.8	120	
25	4-16-65		308	16.5	710	
26	4-17-65	O.C.	439	36.0	325	
a	10-5-62	S.C.Is.	205	--	---	
b	11-2-62	S.C.Is.	529	--	---	
c	3-13-63	N.C.	574	14.4	160	
d	4-9-63	N.C.	446	18.0	16.0	
e	8-15-63	R.C.	221	--	640	
f	12-15-63	C.Is.	221	21.5	70	
g	12-16-63	C.Is.	820	--	---	
h	11-20-64	C.Is.	374	--	45	
i	11-21-64	C.Is.	241	--	70	
j	4-17-65	C.Is.	---	--	---	N.A.

O.C. Orange County
S.C.Is. San Clemente Island
N.C. Newport Canyon
R.C. Redondo Canyon
C. Is. Santa Catalina Island
S.B. Santa Barbara
V Ventura
Ox Oxnard
N.A. Not Applicable

Table 2
(Cont.)

No.	Date and Location		$\bar{W}$ (cm/sec)	$\bar{U}$ (cm/sec)	$\bar{S}$ (gm/cm^4) x 10^{-6}	$\bar{Q}$ (cm^3/sec)
II Point-Source Plume Experiments:						
1	6-27-63	O.C.	309	16.5	---	N.A.
2	6-28-63		310	17.5	640	
3	9-11-63		380	20.6	470	
4	9-12-63		335	9.3	720	
5	11-9-63		---	7.7	610	
6	11-10-63		82	13.8	1230	
7	2-8-64	O.C.	288	7.2	380	N.A.

No.	Date and Location		$\bar{W}$ (cm/sec)	$\bar{U}$ (cm/sec)	$\bar{S}$ (gm/cm^4) x 10^{-6}	$\bar{Q}$ (cm^3/sec) (x 10^4)
III Large-Scale Plume Experiments:						
1	6-17-64	O.C.	397	16.4	510	430
2	12-16-64	O.C.	484	19.2	465	430
3	6-8-65	S.B.	772	13.4	135	35
4	1-27-66	S.B.	890	8.3	---	36
5	1-28-66	S.B.	155	10.3	---	37
6	3-17-66	S.B.	349	6.2	---	35
7	3-18-66	S.B.	670	9.2	---	35
8	5-13-66	S.B.	644	15.0	---	35
9	6-10-66	V (-)	278	10.8	---	10
10	8-15-66	V (-)	669	10.3	---	15
11	8-16-66	V (-)	309	12.8	---	15
12	8-17-66	V (-)	493	8.3	---	15
13	10-13-66	Ox	470	9.2	---	79
14	11-10-66	V	493	8.3	---	10
15	11-11-66	V	541	8.3	---	10
16	2-23-67	V	314	10.4	356	11
17	2-24-67	V	203	13.4	---	11
18	3-23-67	Ox	790	5.7	---	33
19	4-27-67	V	603	9.2	---	11
20	4-28-67	V	340	12.8	---	11
21	6-7-67	Ox	736	20.2	---	63
22	6-8-67	Ox	726	12.8	---	64
23	8-2-67am	V	304	6.7	---	10
24	8-2-67pm	V	844	4.1	---	10
25	8-3-67	V	267	18.9	---	10
26	2-28-68	S.B.	515	10.4	---	42
27	2-29-68	S.B.	206	5.7	---	44
28	3-20a-68	S.B.	412	6.7	---	42
29	3-20b-68	S.B.	411	6.7	---	42
30	3-21-68	S.B.	337	4.6	---	44
31	6-11a-68	V	601	12.8	---	10
32	6-11b-68	V	684	6.2	---	10
33	6-12-68	V	172	13.4	---	10
34	9-4-68	V	878	6.7	84.9	9

Table 3
Summary of
Diffusion Parameters

No.	$\bar{K}_x$ (cm^2/sec)	$\bar{K}_y$ (cm^2/sec)	$\bar{K}_z$ (cm^2/sec)	$\bar{\sigma}_x$ (cm)	$\bar{\sigma}_y$ (cm)	$\bar{\sigma}_z$ (cm)	σ_{xo} (cm)	σ_{yo} (cm)	σ_{zo} (cm)
I. Dye Patch Experiments:									
2	11612	3614	2.79	7276	4859	85.3	1140	2003	28.76
3	2750	1839	0.72	3920	3737	65.5	2320	2326	43.75
4	5109	5082	1.93	6392	4907	176.2	1350	--	109.47
5	11984	5528	0.15	6559	5233	77.1	1530	--	63.21
6	---	--	0.65	--	--	76.8	--	--	39.86
7	---	--	1.76	--	--	159.1	--	--	63.94
8	25548	652	0.63	17413	2377	96.0	2120	1372	28.60
10	13471	596	0.65	12034	3079	108.2	750	1341	46.53
11	12030	1932	0.9	8391	3210	100.9	1840	1362	39.74
12	10405	883	3.36	8565	3022	221.2	1610	985	110.74
13	5546	1486	4.93	5090	4450	185.3	606	2743	84.03
14	---	--	--	--	--	--	--	--	--
15	12542	1291	2.65	8050	4161	150.9	860	2682	69.51
16	9012	1291	0.55	5785	4161	79.6	1455	2682	46.23
17	16908	2973	10.96	11232	5456	265.2	8110	3840	137.33
18	26755	2945	--	10592	4505	--	1062	2304	127.14
19	20903	397	0.55	11345	1806	95.7	1350	1417	72.77
20	2248	1013	0.96	5243	1964	162.2	3360	1140	138.01
21	49330	2090	0.73	11220	3741	124.1	1250	1996	97.34
22	4273	563	1.53	5480	2135	132.9	1150	796	73.59
23	28056	1812	0.53	12040	4286	81.7	3050	2384	53.67
24	39947	--	28.43	8281	--	297.5	2330	--	61.56
25	4924	--	0.23	5148	--	62.2	1060	--	49.24
26	5648	561	2.55	4788	1662	109.1	2560	591	67.19
a	2787	118	2.41	4657	793	114.9	1880	384	37.70
b	3177	520	21.2	3374	1670	417.9?	896	841	97.82
c	58992	1933	8.58	16810	3213	253.0	2280	1366	121.16
d	---	251	4.9	--	1220	--	--	756	--
e	25597	696	0.37	14030	2220	95.4	843	680	68.83
f	3289	391	3.93	4993	1770	218.8	663	728	111.57
g	---	--	12.82	--	--	363.3	--	--	192.77
h	2508	240	4.43	3962	1450	177.1	670	759	51.46
i	5017	3437	3.1	5883	5730	185.3	640	2408	145.86
j	2044	4060	1.07	3463	3810	54.3	2390	2819	37.34

Table 3
(Cont.)

No.	$\bar{K}_x$ (cm^2/sec)	$\bar{K}_y$ (cm^2/sec)	$\bar{K}_z$ (cm^2/sec)	$\bar{\sigma}_x$ (cm)	$\bar{\sigma}_y$ (cm)	$\bar{\sigma}_z$ (cm)	σ_{xo} (cm)	σ_{yo} (cm)	σ_{zo} (cm)
II. Point-Source Plume Experiments:									
1	N.A.	11935	0.38	N.A.	6900	76	N.A.	N.A.	N.A.
2		19530	--		8350	--			
3		18600	0.4		--	50			
4		1330	0.66		1880	78			
5		9052?	--		?	--			
6		4185?	0.08		?	46			
7	N.A.	4201?	0.23	N.A.	?	96	N.A.	N.A.	N.A.
III. Large-Scale Plume Experiments:									
1	N.A.	---	1.5	N.A.	--	101	N.A.	--	37
2		26000	1.9		26400	144		10302	76
3		1210	37.2		3690	247		3048	--
4		---	--		--	--		--	--
5		---	--		--	--		--	--
6		---	--		--	--		--	250
7		5350	22.3		8300	514		6300	241
8		6140	9.1		6640	174		4980	--
9		7400	--		4660	--		3040	143
10		---	--		--	--		--	305
11		585	0.26		2440	107		1052	101
12		1080	0.77		--	137		--	107
13		3830	1.0		7760	177		4770	311
14		1990	0.72		7760	244		1425	213
15		1290	6.7		3020	293		1100	183
16		2700	--		5800	--		1285	247
17		---	--		--	--		--	180
18		---	--		--	--		--	--
19		409	20.4		2010	546		825	442
20		632	0.81		2920	207		810	189
21		1910	16.7		3580	299		1530	686
22		1020	7.9		2900	451		1530	411
23		---	--		--	--		2380	--
24		---	--		--	--		--	320
25		3610	0.84		4370	522		2190	180
26		1750	--		6900	--		8170	503
27		---	--		--	--		--	180
28		---	--		--	--		--	--
29		---	--		--	--		--	--
30		---	--		--	--		--	226
31		---	--		---	--		--	149
32		---	--		--	--		--	--
33		---	--		--	--		--	--
34	N.A.	---	18.02	N.A.	--	--	N.A.	--	235

Table 4
Outfall Characteristics

Location	Length (m)	Diameter (cm)	Diffuser
Orange County (-)	2030	198	None
Orange County	2030	198	305 m long, 15 cm ports @ 4.88 m spacing
Oxnard	2070	147	117 m long, 15 cm ports @ 3.7 m spacing
Santa Barbara	1140	114	91 m long. 15 cm ports @ 3 m spacing
Ventura (-)	975	89	None
Ventura	975	89	17 m long, 15 cm ports @ 2.4 m spacing

Note (-) = without diffuser
Diffusers were added to the Orange County and Ventura outfalls during the course of the investigation.